Wavelet based recursive identification of modal parameters

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Abstract

This paper presents a recursive method of modal parameters identification based on operational measurements, dedicated for non-stationary systems. Decoupling signal components procedure allowed to reduce the signal model and simplified the process of modal parameters estimation. Adaptive method of filtering simplified the process of wavelet function selection. Presented method utilize Continuous Wavelet Transform (CWT) with Complex Morlet Wavelet function. Thanks to reduction of model order, estimation of modal parameters can be performed using relatively simple mathematical formula. This approach significantly reduces demand for computing power which have a direct impact on system costs and modal parameter estimation time. The method has been tested on numerical models and applied for real data.

1 Introduction

The operational modal analysis (OMA) is widely used in civil, mechanical and aerospace engineering communities and applied to identify the modal parameters of such structures as buildings, towers, bridges, offshore platforms, airplanes, etc. [1]. However, the classical OMA's techniques have some limitations, among which the following are the most important [2]: the structure is assumed to be linear, the structure is time invariant, the structure is observable and in the system of interest damping is small or proportional. Due to this assumptions, results which can be achieved with modal technique are an approximation of the real structure behavior, but still, they are good enough to be applied in diagnostics, monitoring, control, etc.

In practice, many engineering structures like aircrafts, traffic-excited bridges, robots, rotating machinery working with varying speed, cranes and many others should be treated as non-stationary systems. This means that at least one of the assumptions is not satisfied in this case. The classical techniques of OMA do not allow for variation of the eigenvalue matrices [3], [4]. Therefore, the techniques cannot be directly applied to identification of non-stationary systems' modal parameters. Additionally, there are practical problems associated with the implementation of the OMA methods included: length of the data required for analysis, duration of the estimation procedure, necessity of an experienced operator's intervention in order to select the correct results from a set of solutions or influence of estimation procedure sequence in case of data obtained during a series of partial experiments (runs or set-ups).

The most frequently used solution for application OMA to identification of non-stationary system is the "quasistationarity" assumption. It is assumed that the system is stationary within given time interval. Unfortunately, this solution requires a compromise between the ability of the algorithm to keep up with modal parameters changes and the quality of results (small number of samples). Another approach is the use of recursive methods of identification. In this case, the eigenvalue matrix can be estimated recursively for every sample.

Despite the above mentioned limitations, the OMA techniques can be adapted to identification of non-stationary systems. Various varieties of OMA are widely used in non-stationary condition and can be applied for identification e.g. systems containing rotating parts (such as turbines, helicopters, engines, etc.) where the assumption of broadband of excitation is not fulfilled [5], structures with variable mass [6] and geometry

[7], systems with closely spaced modes [8], large structures [9] and systems with varying boundary conditions [10,11].

This paper focus on application of adaptive approach to wavelet filtering process. The use of wavelet filter allows to: decoupling individual frequency components of the signal, reduction of the signal model and simplified the process of the modal parameters estimation. The adaptive approach simplified the process of wavelet function selection and enables changes/adaptation of wavelet frequency during identification process.

The paper is organized in the following way. Section 2 describes the method for model parameters estimation. Section 3 presents the adaptive wavelet based signal filtration. In Section 4 the algorithm is presented. The next sections contains the results of verification of the method on simulated data. Identified parameters are compared with the results obtained by using a non-adaptive formula of presented algorithm.Erreur ! Signet non défini.

2 Identification method

The proposed algorithm consists of three main parts. In the first step the signal is decomposed by wavelet transform. In second step model parameters are estimated. In third step a modal determined. Organization of the algorithm shown in Figure 1.

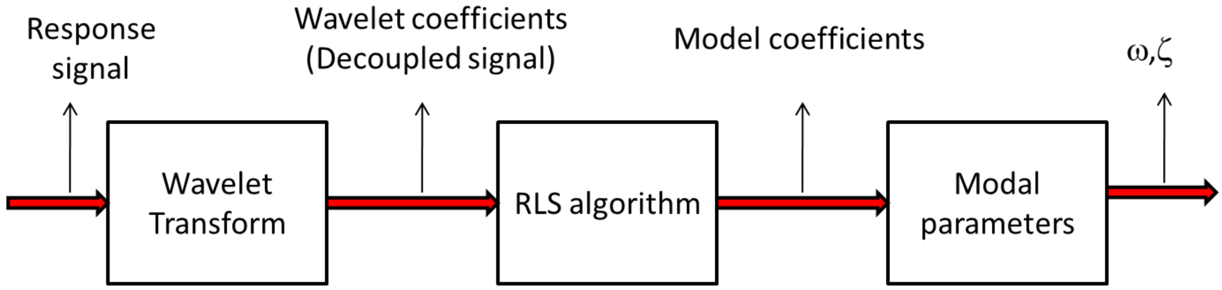

Figure 1 : Organization of the algorithm

2.1 Adaptive wavelet filtering

The wavelet analysis is a method of signal decomposition. As a result of the wavelet analysis, in contradiction to the Fourier transform, elementary signals – so called wavelets – are obtained. Wavelet functions are continuous, oscillated with various duration times and spectrums. From the mathematical point of view, a continuous wavelet transform (CWT) of a signal x(t) can be defined as

$$(W_g x)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) g^* \left(\frac{t-b}{a} \right) dt$$

where *b* is a translation (displacement) representing region, *a* is dilatation (expansion) or scale parameter, *g(t)* is the basic wavelet function. Wavelet filtering decomposes particular signal frequency components with resolution depending on the wavelet function parameters [12]. The selection of parameters of the wavelet function requires some compromise between the quality of filtration in frequency and time domain. After decoupling, every component of the signal can be analyzed separately. This significantly decreases computational effort because model order of decoupled component is low. Besides, tracking all natural frequencies and damping ratios of the system is not always demanded. An example of this can be flight flutter tests, where very often only a specified number of frequency and damping ratios (directly responsible for the flutter phenomenon) is tracked. It also reduces the demand for computational power which increases applicability of the method and makes the real-time implementation process easier. Schematically, the process of frequency component decoupling for stationary signal is shown in Figure 2.

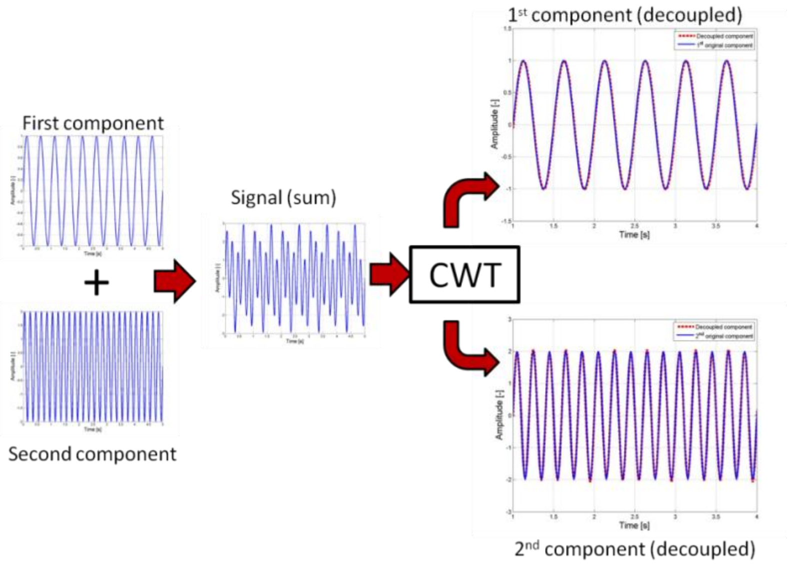

Figure 2. Wavelet based signal filtering.

This property has been repeatedly used to identify modal parameters for both stationary and non-stationary systems. The major inconvenience of using the non-adaptive version of a wavelet filter for modal parameters identification process is the constant filtration bandwidth for assumed wavelet function. For large changes in system parameters and use of a narrow band filter, decomposition process can be performed for another(next) signal component or decomposition can be applied for frequency band where there are no frequency components. Then, the obtained results can be only a filter response This problem is presented in Figure 3a. In turn, application of broadband wavelet filter causes decrease in time domain resolution, according to Heisenberg relation.

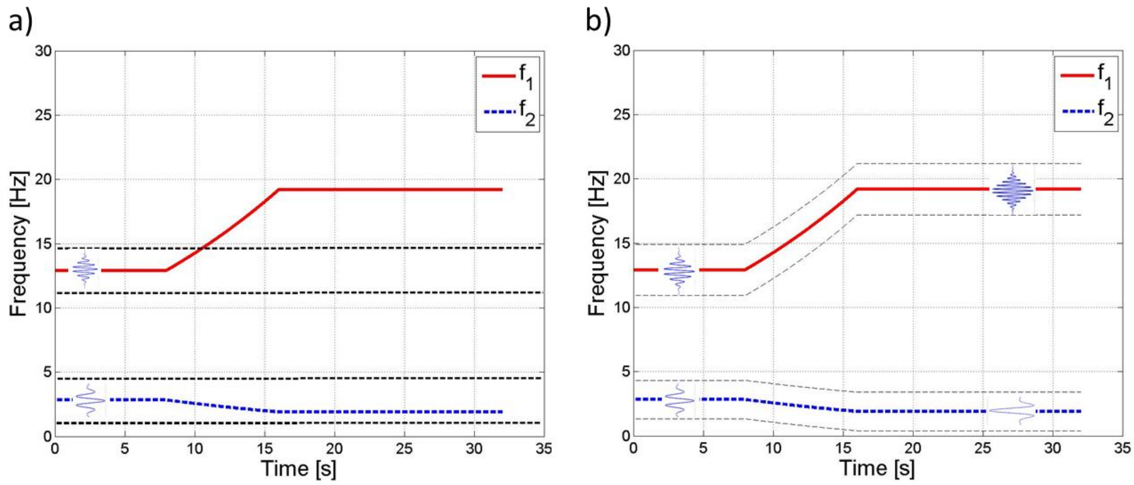

Figure 3. Comparison of Non-Adaptive (constant scale parameter) (a) and adaptive (variable scale parameter)(b) wavelet filtering conception. The f_1 and f_2 are tracked frequency, related to scale parameter a.

The solution of the constant filter bandwidth problem can be making the bandwidth parameter conditional on the identification process. This requires determination of the wavelet parameters and different wavelet functions g_i for different discrete time moments i:

$$\left(W_{g_i} x_i\right)(a_i, b_i) = \frac{1}{\sqrt{a_i}} \int_{-\infty}^{+\infty} x_i(t) g_i^* \left(\frac{t_i - b_i}{a_i} \right) dt$$

Determination of the wavelet functions, which allows filtration of the given frequency component, requires defining the scale parameter associated with the frequency by the formula [13]:

$$f_i = \frac{T_s}{a_i}$$

where T_s is the sampling time and f_i is the frequency corresponding to scale parameters a_i. A change of the scale parameter a_i allows change of the wavelet filter frequency. Schematically, this process is shown in Figure 3b.

2.2 Recursive Least Square algorithm

Method of estimation model coefficients based on RLS algorithm. Schematically the algorithm is presented in Figure 4

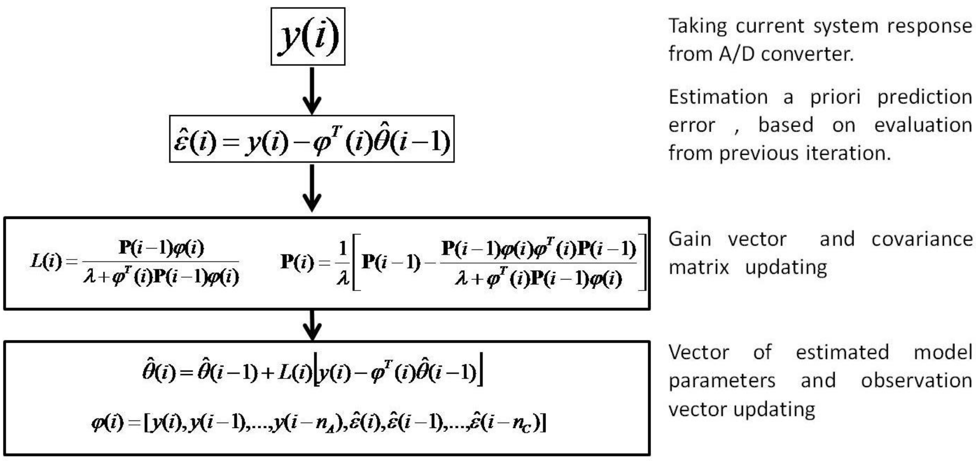

Figure 4. Organization of Recursive Least Square method.

where $y(i)$ is current system response signal, $\hat{\varepsilon}(i)$ is estimated a priori prediction error, based on previous iteration

$$\hat{\varepsilon}(i) = y(i) - \varphi^T(i)\hat{\theta}(i-1)$$

$\varphi(i)$ is regressor vector, $\theta(i)$ is vector of model parameters, $L(i)$ is gain vector and $P(i)$ is covariance matrix given as

$$L(i) = \frac{\mathbf{P}(i-1)\varphi(i)}{\lambda + \varphi^T(i)\mathbf{P}(i-1)\varphi(i)}$$

$$\mathbf{P}(i) = \frac{1}{\lambda}\left[\mathbf{P}(i-1) - \frac{\mathbf{P}(i-1)\varphi(i)\varphi^T(i)\mathbf{P}(i-1)}{\lambda + \varphi^T(i)\mathbf{P}(i-1)\varphi(i)}\right]$$

where λ - forgetting factor. For , $i = 0$, $P(0) = \delta I$,where δ - is a large natural number , for example 10^6 , I - unit matrix.

As a result of RLS algorithm the vector of model parameters is obtained

$$\theta^T(k) = [-a_1, \ldots, -a_{n_A}, 1, c_1, \ldots, c_{n_C}]$$

where n_A and n_C are model order, $a_1, \ldots, -a_{n_A}, c_1, \ldots, c_{n_C}$ are model coefficients.

3 The algorithm

The adaptive identification algorithm consists of two main parts. The core of the algorithm is RLS algorithm – responsible for model parameters estimation. The input data are filtered using wavelet transform, which allows both reduce the model order and estimate the modal parameters based on analytical formulas. An integral part of the identification process is an adaptation of wavelet filter that allows to tune the filter characteristics to the current value of the frequency.

3.1 The adaptation process

The adaptation process is performed by comparing the current frequency of wavelet function (*a* parameter) and frequency estimated by the RLS algorithm. If the absolute value of the difference of the two frequencies is contained within the assumed range (δ), the identification process is continued without changes. If the difference is greater than assumed, the frequency of wavelet function (scale parameter) is changed to a value corresponding to the frequency estimated from the RLS algorithm. Schematically, the process of adaptation and the diagram of the method with adaptive wavelet filtering is presented in Figure 5, where f_e, f_w and δ are respectively: current estimated frequency, frequency corresponding to scale parameter (wavelet frequency) and adaptation step. The adaptation step allows to reduce the instantaneous growth of covariance matrix values at the moment of change of the wavelet filter parameters [14,15]. The range of this parameter is determined as much smaller than the filter bandwidth (1-5% of bandwidth) to guarantee the correct signal components decoupling.

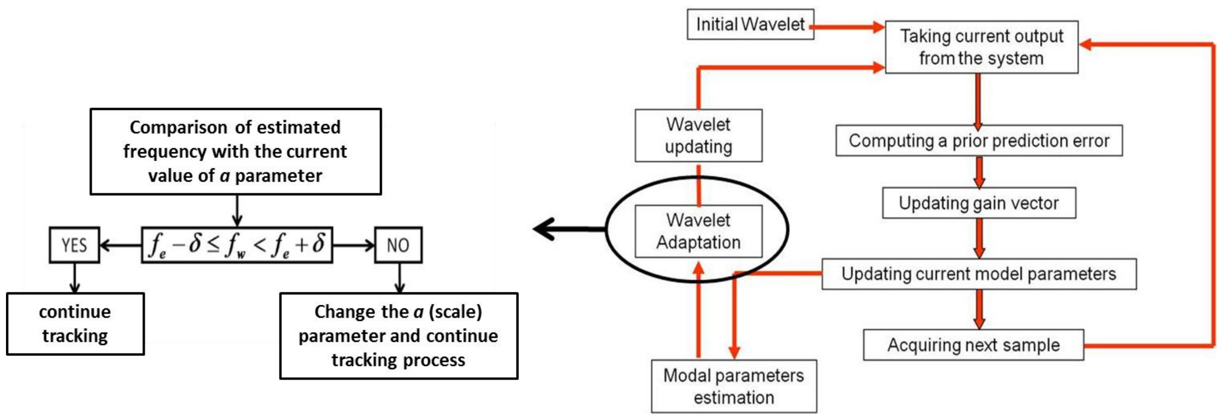

Figure 5. Organization of proposed adaptive wavelet filtering procedure.

3.2 Modal parameters estimation

Wavelet decoupling allows to separation particular signal components. The result is that model order of the signal is known and equal two. For the second order signal model it is possible to assign analytical formulas that describe dependences between model and modal parameters [25]

$$\omega = \frac{1}{2}\frac{\sqrt{\frac{\ln\left(\frac{1}{2}\sqrt{a_1 + |a_1 - 4a_2|}\right)}{T_s} + \frac{\arctan\left(\frac{1}{2}\sqrt{a_1 + |a_1 - 4a_2|}\right)}{T_s}}}{\pi}$$

$$\zeta = -\frac{\ln\left(\frac{1}{2}\sqrt{a_1^2 + \left|a_1^2 - 4a_2\right|}\right)}{T_s\sqrt{\frac{\ln\left(\frac{1}{2}\sqrt{a_1^2+\left|a_1^2-4a_2\right|}\right)^2}{T_s^{\,2}} + \frac{\arctan\left(\frac{1}{2}\sqrt{\left|a_1^2-4a_2\right|},-\frac{1}{2}a_1\right)^2}{T_s^{\,2}}}}$$

where: ω - natural frequency, ζ - damping ratio. When the analytical formula for modal parameters calculation is assigned, there is no required to find roots of the characteristic polynomial estimated from the RLS algorithm.

4 Verification of the algorithm – numerical non-stationary model

The two degree of freedom non-stationary system was modelled. The stiffness coefficient for mode 1 was changed during simulation according to equation

$$k_1 = \begin{cases} 26000 & for \quad 0 < t \le t_1 \\ 26000 * e^{0,001}t & for \quad t_1 < t \le t_2 \\ 59722 & for \quad t_2 < t \le t_3 \end{cases}$$

An example of time history of the system response and the stiffness parameters are presented in Figure 6.

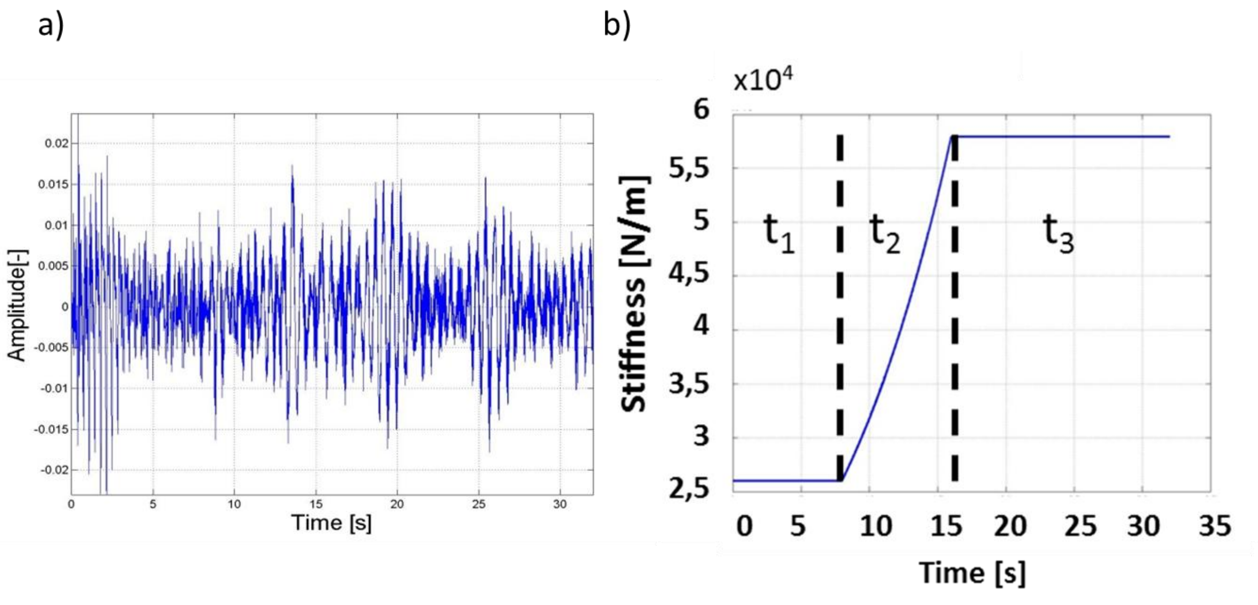

Figure 6: a)System response for white noise excitation, b) stiffness changes for mode 1

Using procedures described in the section 3, the identification process was performed. Both adaptive and non-adaptive algorithms had the same initial parameters applied (wavelet function and forgetting factor). Wavelet function parameters were selected randomly. Comparisons of the identification results using non-adaptive and adaptive filtering are presented in Figure 7 where identified values of damping ratio and natural frequencies of the system are presented.

a) b)

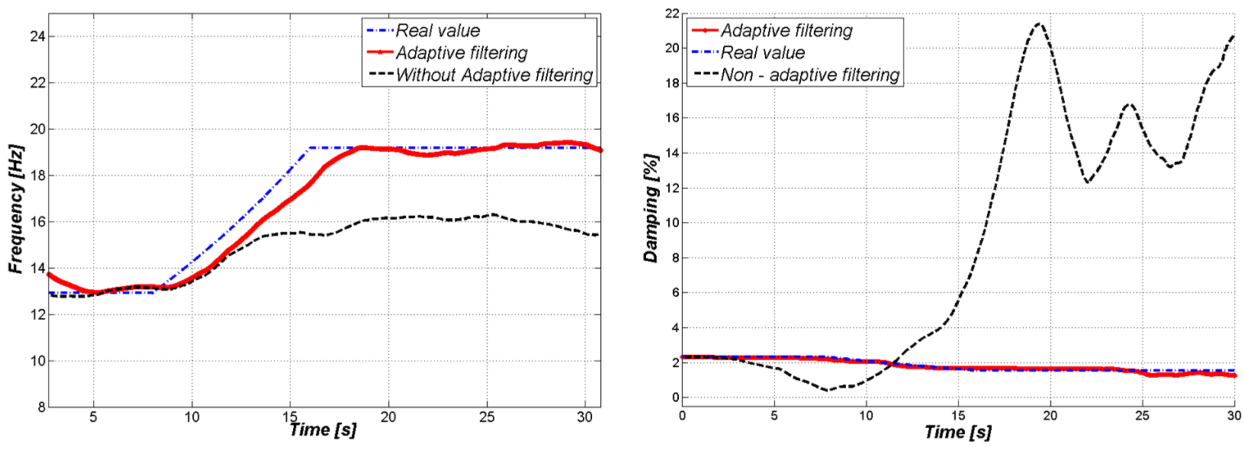

Figure 7. Comparison of identification results for mode 1: a) natural frequency, b) damping ratio

As can be noticed, the method with adaptive wavelet filter gives better results for both natural frequency and damping ratio. There are two conclusions arising from the performed test: adaptive wavelet filtration enables tracking of modal parameters and the process of initial wavelet selection is not a critical part of the algorithm, unlike in the non-adaptive method (as described in previous work of the authors).

5 Experimental verification of formulated procedures

Two experiments on real objects were performed. First, the real time identification of the modal parameters of a system with variable stiffness has been conducted. Next, the algorithm has been used for identification of the modal parameters of ISKRA air jet during a flight.

5.1 Identification of modal parameters of system with variable stiffness.

The test bed was build out of the three main parts: frame, cart and two metal bellows. Between the cart and the frame there are two metal bellows mounted. The friction between the cart and the frame has been eliminated thanks to air bearings. Experimental setup of experiment is presents in Figure 8.

a) b)

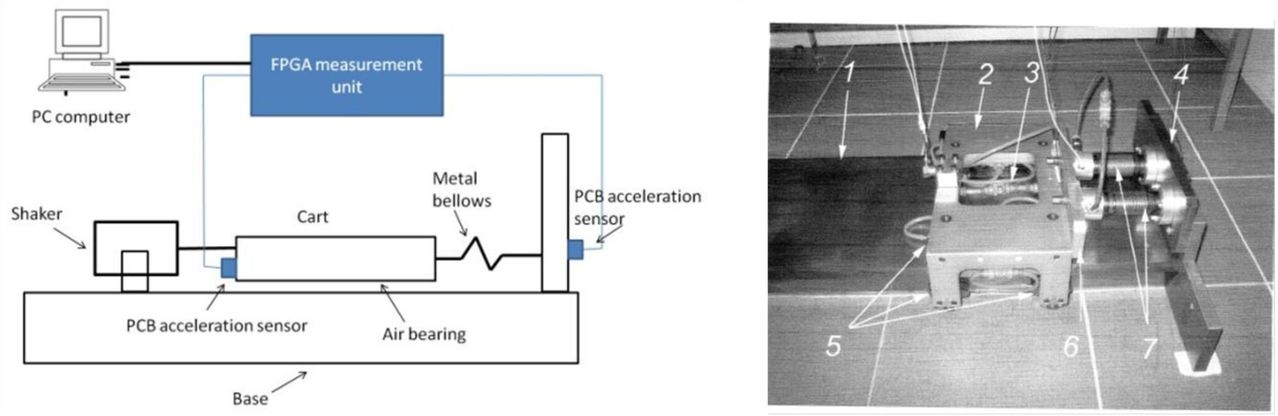

Figure 8 : Experiment arrangement: a) scheme of the laboratory stand, b) equipment for experiment prepared to free vibration run test 1. frame, 2. cart on the air bearings, 3. air bearing hoses, 4. metal bellows bracket mounted to the frame, 5. air bearings, 6. metal bellows bracket mounted to the sliding cart, 7. two metal bellows.

An electromagnetic shaker and a signal generator were used to excite the structure. The white noise signal was used as an excitation signal. During the experiment, the pressure in metal bellows was changed. As a result of system stiffness' changes, the natural frequency of the system was shifted. The time history of system response, the result of the natural frequency identification and the adaptation process of the wavelet function are presented in Figure 9. Additionally, wavelet adaptation process was presented in Figure 9b (dashed line).

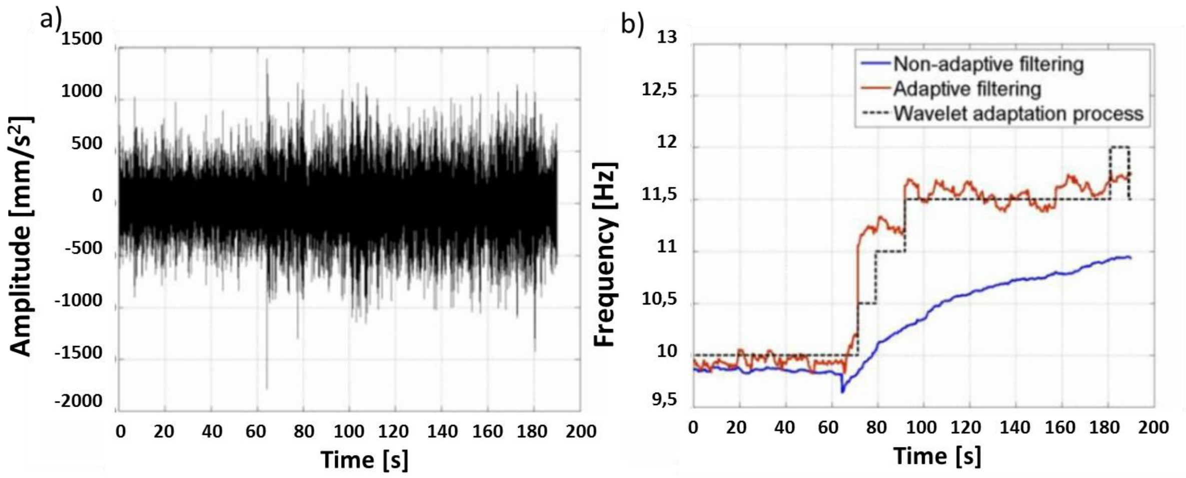

Figure 9. System response (a), Comparison of identified natural frequency of the system (b).

Also in this case, the results obtained with the use of adaptive methods are considerably better than the ones obtained by the non-adaptive wavelet filtering. The algorithm reacts much faster to the changes of the natural frequency of the system. Both algorithms were run with the same initial parameters.

5.2 Modal parameters identification of Iskra air jet during a flight.

TS-11 Iskra "is a two-seater, mid-wing monoplane" jet. The study of the in-flight modal characteristics of the aircraft was based on ten accelerometers, arranged as shown in Figure 10a. During the flight test the aircraft was accelerated to a speed exceeding the maximum speed (during the dive). An example of a system response acquired by sensor 7 is presented in Figure 10b.

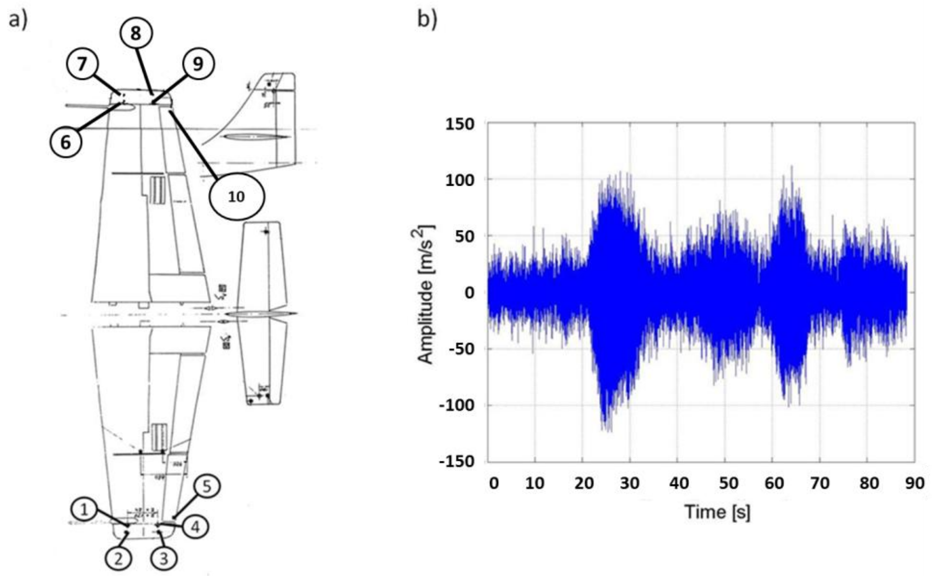

Figure 10. In-flight test: a) Measurement points for in-flight test, b) Example of response signal for sensor 7.

Analysis was perform for mode 27Hz which has the biggest influence on the generation of flutter phenomenon. The comparison of the results estimated by real-time adaptive algorithm with the results estimated by off-line classical non-adaptive RLS algorithm with band-pass filtering is presented in Figure 11.

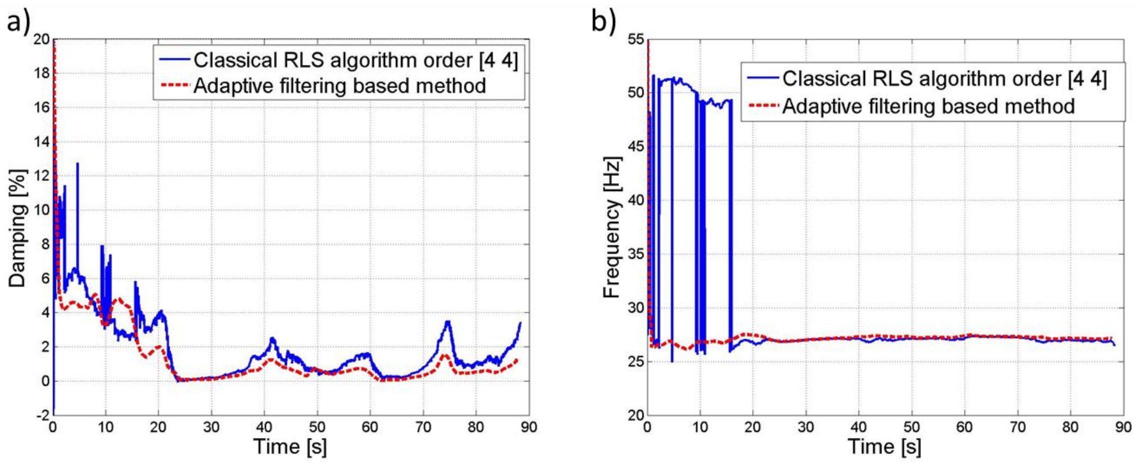

Figure 11. Comparison of result: a) damping ratio, b) frequency

The comparison of the results shows that both algorithms give similar results with exception of the initial phase of identification process (Figure 11b) where damping ratio value is relatively large. It is worth mentioning that the results of the classical RLS algorithm were obtained in off-line mode, with band-pass filtration.

6 Conclusions

Application of the adaptive wavelet filtration to recursive identification of modal parameters have been investigated. The performed test confirmed that the wavelet transform is a useful tool to support the identification process of a non-stationary systems. The adaptive wavelet filtration allows to separate the signal frequency components and enables reduction of model order of analyzed signal. This approach significantly reduce the computation time of modal parameters thanks to the use of analytical formulas for damping ratio and natural frequencies and facilitate the hardware implementation of the algorithm. The algorithm also allows to determine the confidence intervals of modal parameters, which gives the possibility to assess the quality of results. All performed tests showed that the adaptive wavelet filtration method combined with the RLS algorithm gives satisfactory results and works much better than non-adaptive version of the algorithm. This does not disqualify the non-adaptive formula of the algorithm. In the authors' previous works this non-adaptive approach and its hardware implementation was successfully used for real-time identification of modal parameters of non-stationary systems [14; 15] and enable to track tens of natural frequency in real-time (assuming maximum sampling frequency of the signal equals 200Hz). In this case the process of selecting the initial wavelet function and the forgetting factor had to be performed very precisely.

Acknowledgments – additional information

References

[1] Uhl T., Lisowski W.,Kurowski P. In-operation modal analysis and its application. Krakow : AGH University of Science and Technology, 2001.

[2] Piersol Allan G i Paez Thomas L. Harris' shock and vibration handbook. New York : McGraw-Hill, 2010.

[3] Liu, Kefu. Extension of modal analysis to linear time-varying systems. Journal of Sound and Vibration. 1999, Vol. 1, 226, pp. 149-167.

[4] Liu, K. Identification of linear time-varying systems. Journal of Sound and Vibration. 1997, Vol. 206, 4, pp. 487-505.

[5] Pintelon R., Peeters B., Guillaume P. Continuous-time operational modal analysisnext term in the presence of harmonic disturbances—The multivariate case. Mechanical Systems and Signal Processing. 2010, Vol. 24, 1, pp. 90-105 .

[6] Goursat M., Dohler M., Mevel L., Andersen P. Crystal Clear SSI for Operational Modal Analysis of Aerospace Vehicles. Proceedings of the 28th International Modal Analysis Conference (IMAC). 2010, p. CD.

[7] Spiridonakos M.D., Fassois S.D. Parametric identification of a time-varying structure based on vector vibration response measurements. Mechanical Systems and Signal Processing. 2009, Vol. 23, 6, pp. 2029-2048.

[8] Alessandro Agnenia, Luigi Balis Cremaa and Giuliano Coppotell. Output-only analysis of structures with closely spaced poles. Mechanical Systems and Signal Processing. 2010, Vol. 24, 5, pp. 1240-1249.

[9] Andersen P., Brincker R., Goursat M., Mevel L. Automated Modal Parameter Estimation For Operational Modal Analysis of Large Systems. Proceedings of the 2nd International Operational Modal Analysis Conference (IOMAC). 2007, pp. 299-308.

[10] Vu V.H., Thomas M., Lakis A.A., Marcouiller L. Operational modal analysis of non-stationary mechanical systems by short-time autoregressive (star) modelling. 3rd International Conference on Integrity, Reliability and Failure. July 22-24, 2009, p. CD.

[11] Lisowski, W., Uhl, T., Kurowski, P., Mendrok, K., Góral, G., & Klepka, A. (2004). Example of autonomous parameter estimation procedure and its application to results of modal testing of an airplane. Paper presented at the Proceedings of the 2004 International Conference on Noise and Vibration Engineering, ISMA, 1019-1031.

[12] Uhl T., Klepka A. Application of wavelet transform for identification of modal parameters of nonstationary systems. Journal of theoretical and applied mechanics. 2005, Vol. 43, 2, pp. 277 – 296.

[13] Klepka A., Identification of modal parameters of mechanical systems in non-stationary conditions. PhD Thesis. 2006.

[14] Uhl T, Petko M, Karpiel G, Klepka A. Real time estimation of modal parameters and their assessment. Shock and Vibration. 2008, Vol. 15, 3, pp. 299–306.

[15] Uhl T., Bogacz M. Real time modal model identification and its application for damage detection. Proceedings of ISMA 2004 Conference. 2004, pp. pp. 1065- 1075.M. Haritopoulos, J. Roussel, P. Ravier, *Template Sample File 1 for Full Paper Submission*, Chartres, France, 2013 October 28-29, pp.100-110.