Vibroacoustic source separation using an improved cyclic Wiener filter

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Abstract

Noise radiated by rotating and reciprocating machines is often a mixture of multiple complex sources, the successful reduction of which is a field of intensive research. In this paper an advanced source separation approach is presented, based on cyclic Wiener filtering, which takes into account the cyclostationarity property of the signals.

The aim of the Wiener filter is the separation of noisy measurements into their contributions from the N specific sources and the remaining "noise". Traditionally this can be achieved by using reference signals which are strongly coherent with the sources of interest and uncorrelated with all the other interfering sources and the masking noise.

The Wiener filter can be estimated using the raw signals or only their random part. Moreover, the filter can be underestimated if the Signal-to-Noise Ratio of the reference signals is low, thus leading to the paradox that the level of an extracted source contribution is higher than the overall level. In this study a general strategy is proposed in order to select over which part of the signals (raw or residual) should the filter be estimated. This strategy is based on the number of the available references and the expected number of sources and the link with the multivariable statistical regression. Moreover, in order to increase its robustness, it is proposed to estimate the cyclic Wiener filter using an additional constraint which imposes that the sum of the contributions of the periodic parts of each source equals the overall periodic part as is calculated by the synchronous averaging procedure. This produces a new estimator of the Wiener filter, obtained from a constrained least square optimization.

The proposed method is applied on vibroacoustic signals captured on a test rig in order to quantify the contributions of "hydraulic noise" (originating mainly by four hydraulic pumps) and "mechanical noise" (originating from the various rotating parts of the engine).

1 Introduction

The interior and exterior noise levels consist a very competitive factor in the market of modern vehicles and machinery. As a result there is increasing demand for developing quitter equipment. Modern machinery is very complex and the noise emitted is finally the combination of several sources. Since the noise produced is of great concern, efforts are being made by the manufactures to develop tools that allow the accurate quantification, separation and prediction of the effects of the sources.

The sources are usually both spectrally and temporally overlapping. Therefore two main approaches towards solving the problem have been presented, the separation methods based on a priori knowledge of the noise ([1], [2], [3]) (such as provided by a reference signal) and the separation methods based on the statistical independence of the noise sources (blind source separation methods). These two approaches have been applied intensively to the domain of diesel engines. Antoni et al in [4] and El Badaoui et al in [5] took advance of the characteristics of cyclostationarity in order to perform noise source separation by means of the cyclic Wiener filter. The use of Wiener filter was also explored in [6]. MIMO system modelling was used in [7] in order to estimate the noise transfer function of an engine. On the other hand, in [8] blind source separation methods were used in order to recover signals of different physical sources.

This paper is organized as follows. In section 2, the Wiener filter is briefly described. In section 3 a general strategy is proposed in order to select the part of the signals (raw or residual) which should be used for the filter estimation. In section 4, a new constrained Wiener filter is proposed, while in section 5 an approach in order to combine the classic and the constrained filter is presented. The experimental results, obtained by the application of the new approach to a test rig, are demonstrated in section 6. The paper closes with the conclusion in section 7.

2 Estimation of classical Wiener filter

The Wiener filter was introduced by Wiener in 1950s in order to denoise a corrupted stationary signal. Its aim is to separate a noise measurement y(t)

$$y(t) = x(t) + b(t)$$

into its contribution x(t) from a specific source and the remaining "noise" b(t). Traditionally this can be achieved by using a reference signal r(t) which is strongly coherent with the source of interest and uncorrelated with all other interfering sources and masking noise embodied in b(t). The x(t) may be therefore estimated by the filtering operation:

$$\hat{x}(t) = \sum_{\tau} h(\tau) r(t-\tau) \tag{1}$$

As illustrated in Fig. 1, the best linear filter h(τ) can be estimated by minimizing the least square error:

$$h(\tau) = \arg\min_{h(\tau)} \mathbb{E}\left\{\left|y(t) - \hat{x}(t)\right|^2\right\}, \forall t \tag{2}$$

Following the Parseval theorem the filter h(τ) corresponds to the transfer function H(f) which minimizes respectively:

$$H(f) = \arg\min_{H(f)} \mathbb{E}\left\{\left|Y(f) - \hat{X}(f)\right|^2\right\} \tag{3}$$

Finally the classical Wiener filter in the frequency domain is estimated by:

$$H(f) = \frac{S_{yr}(f)}{Srr(f)} \tag{4}$$

where Syr, Srr are respectively the cross spectral and autospectral power densities

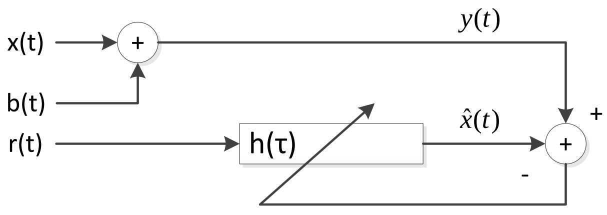

Figure 1: Principle of Wiener filtering

3 Interpretation of Wiener filter and strategy proposition

The Wiener filter can be interpreted in the light of the statistical regression, as it estimates the transfer function H(*f*) which minimizes the mean square error at each frequency (3). The general model of the linear regression is:

$$Y(f) = C(f) + H(f)R(f) + B(f) \tag{5}$$

where C(*f*) is the y-intercept, H(*f*) the slope of the regression, R(*f*) the reference and B(*f*) the model of noise. The regression line should pass from the origin of the axes as a result C(*f*)=0. The model of noise respects two basic hypotheses:

a) The perturbations B(*f*) are independent and identically distributed and
b) The perturbations B(*f*) follow the normal distribution.

A third hypothesis should be added:

c1) The perturbations B(*f*) have a mean value equal to zero (E{B(*f*)}=0).
Or
c2) The perturbations B(*f*) present a periodic part.

As mentioned in previous session, the classical and the constrained Wiener filter can be estimated using the raw signals or only the residual part of the signals. Based on the third hypothesis and taken into account that $E\{B^R(f)\}=0$, a strategy can be proposed for the selection of the part of the signals on which the filter should be estimated. The strategy can be described by two scenarios:

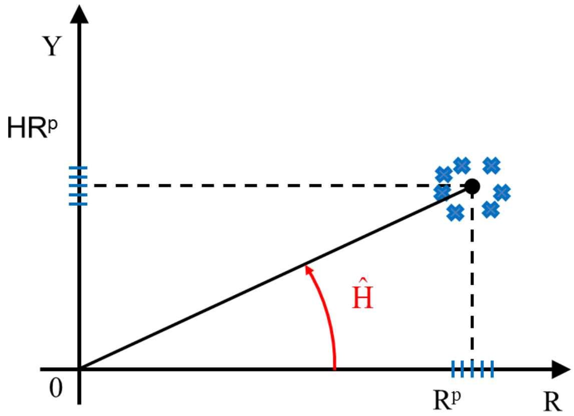

Figure 2: The Wiener filter presented as the slope of the line of regression estimated for each frequency using the raw signals

- **Scenario I:** $B(f)=B^R(f)$

In this scenario the number of references is equal to the number of sources. The slope of the regression line corresponds to an estimation of the filter G ($G=\hat{H}$) (Figure 2). In the presence of intensive harmonic components, the cloud of points is localised around the point $\{R^P, HR^P\}$ leading to the correct and appropriate filter G. The filters should be estimated using the raw signals.

- **Scenario II:** $B(f)=B^R(f)+B^P(f)$

In this scenario the number of references is smaller than the number of sources. The noise presents a periodic part which could come from an unknown source or from a source for which a reference is not available. In this case, using the raw signals in order to calculate the filter G would lead to an estimation of the filter H_1 (which passes through the origin, Figure 3) and not to an estimation of the true filter H as is demonstrated at Figure 4. As a result the residual signals should be used for the estimation of the filter.

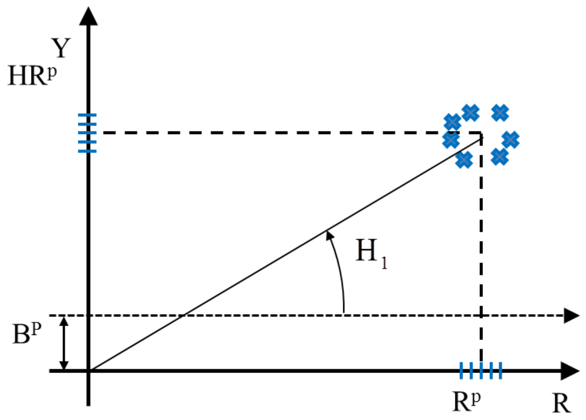

Figure 3: The estimated Wiener filter at the Scenario II

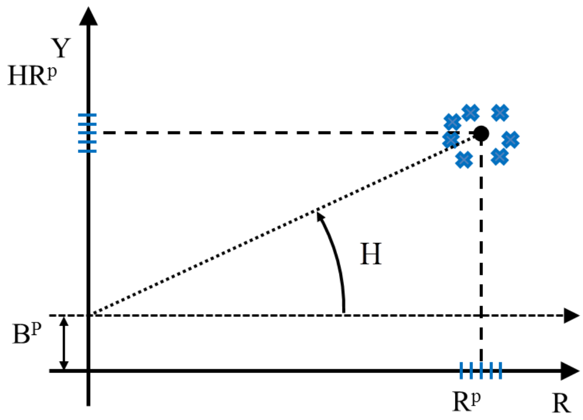

Figure 4: The estimated Wiener filter at the Scenario II

4 Estimation of constrained Wiener filter

In order to improve the estimation of the Wiener filter G, the imposition of a new constraint is proposed:

$$\langle Y \rangle = \underline{\hat{X}}^{P} = \underline{\underline{G}}\,\underline{R}^{P} \tag{6}$$

Practically, the constraint imposes that the estimation of the periodic part of the signal should be equal to the result of the synchronous averaging. As a result, a new filter $\underline{\underline{G}}_{con}$ is constructed and proposed.

The random complex variables $\underline{N}^{R} \sim CN\left(0, \underline{\underline{\Omega}}_{N}\right)$ $\underline{Y}^{R} \sim CN\left(\overline{\underline{X}^{R}}, \underline{\underline{\Omega}}_{N}\right)$ are considered. According to the central limit theorem applied on the Fourier transformation, the random complex variable $\underline{Y}^{R}(f)$ follows a normal probability density:

$$p\left(\underline{Y}^{R}(f)\right) = \frac{1}{\pi^{m}\left|\underline{\underline{\Omega}}_{N}\right|}\exp\left\{-\left(\underline{Y}^{R}(f) - \underline{\underline{G}}_{con}\underline{R}^{R}(f)\right)^{H}\underline{\underline{\Omega}}_{N}^{-1}\left(\underline{Y}^{R}(f) - \underline{\underline{G}}_{con}\underline{R}^{R}(f)\right)\right\}$$

The independent observations $\underline{Y}^{R}(f,k)$, k=*1,...,* *I* of the random variable obtained over I consequent cycles follows a normal joint probability distribution:

$$\left[\underline{Y}^{R}(f;1),\ldots,\underline{Y}^{R}(f;k),\ldots,\underline{Y}^{R}(f;I)\right] = \prod_{k=1}^{I}\left[\underline{Y}^{R}(f;k)\right] =$$
$$= \prod_{k=1}^{I}\frac{1}{\pi^{m}\left|\underline{\underline{\Omega}}_{N}\right|}\exp\left\{-\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)^{H}\underline{\underline{\Omega}}_{N}^{-1}\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)\right\}$$

The principle of maximum likelihood consists in estimating the matrix $\underline{\underline{G}}_{con}$ which maximizes the probability of observation of the data $\underline{Y}^{R}(f)$ over the k=1,..., I measured cycles, i. e. the density, or using a more convenient approach, its logarithm (or which minimizes the negative logarithm) under the constraint:

$$\min_{\underline{\underline{G}}_{con}} -\ln\left[\prod_{k=1}^{I}\frac{1}{\pi^{m}\left|\underline{\underline{\Omega}}_{N}\right|}\exp\left\{-\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)^{H}\underline{\underline{\Omega}}_{N}^{-1}\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)\right\}\right] \Leftrightarrow$$
$$\min_{\underline{\underline{G}}_{con}} \sum_{k=1}^{I}\left[m\ln\pi + \ln\left|\underline{\underline{\Omega}}_{N}\right| + tr\left\{\underline{\underline{\Omega}}_{N}^{-1}\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)^{H}\right\}\right] \Leftrightarrow$$
$$\min_{\underline{\underline{G}}_{con}}\left[tr\left\{\underline{\underline{\Omega}}_{N}^{-1}\sum_{k=1}^{I}\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)\left(\underline{Y}^{R}(f;k) - \underline{\underline{G}}_{con}\underline{R}^{R}(f;k)\right)^{H}\right\}\right]$$

The lagrangian of the minimization problem is introduced and after a number of steps the final result is given by the equation:

$$\underline{\underline{G}}_{con} = \underline{\underline{G}} - \left(\underline{\underline{G}}\,\underline{R}^{P}(f) - \underline{\hat{X}}^{P} \right) \left(\underline{R}^{P}(f)^{H}\, \underline{\underline{S}}_{R^R R^R}^{-1}(f)\, \underline{R}^{P}(f) \right)^{-1} \underline{R}^{P}(f)^{H}\, \underline{\underline{S}}_{R^R R^R}^{-1}(f) \tag{7}$$

Returning to the statistical regression and the graphical representation, the imposition of the new constraint practically sets a constraint at the maximum least square error of the filter estimation as is demonstrated in Figure 5.

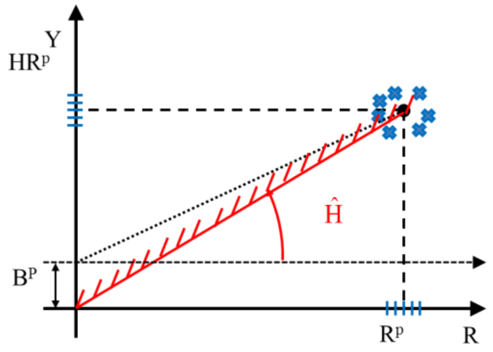

Figure 5: Maximum limit of the possible estimation error of the Wiener filter G.

5 Combination of classical and constrained Wiener filter

In order to improve the separation results, the classical and the constrained Wiener filter are combined in an optimization procedure. The constrained Wiener filter is applied only on the frequencies where the classical Wiener filter leads to an overestimation of the estimated contribution of each source and of the estimated overall noise in comparison with the overall measured noise at the specific frequencies. The procedure is briefly described in the following flowchart (Figure 6) where Syy is the spectral density function of the measured output, EstSyy is the estimation of the spectral density function of the output, $EstSy_iy_i$ is the estimation of the spectral density of each source and Ns is the number of sources.

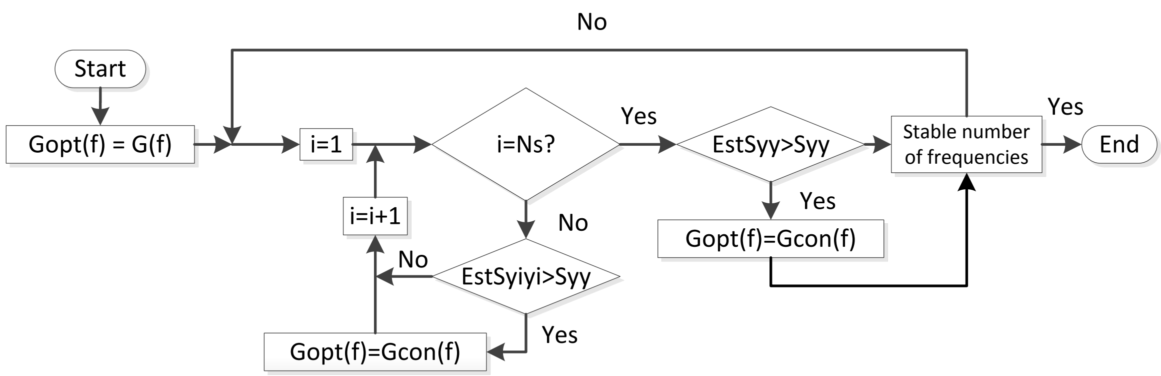

Figure 6: Optimisation algorithm for the estimation of G_{opt} filter

The algorithm scans all the frequencies and finds those frequencies where the application of the estimated Wiener filter G leads to an overestimation either of the estimated spectra of each source EstSyiyi or of the estimated spectra of the overall noise EstSyy. At the specified frequencies the filter G is substituted by the constrained filter G_{con}. The algorithm iterates either till there is no overestimation or till the number of frequencies where overestimation comes up is stabilised. By following this approach a new G_{opt} filter is estimated.

6 Experimental results

The proposed method is applied on vibroacoustic signals captured at a test rig in order to quantify a) the contributions of the "total hydraulic noise" (originating mainly by four hydraulic pumps) and the "mechanical noise" (originating from the various rotating parts of the engine) and b) the contribution of each hydraulic pump. The hydraulic pressures of the pumps are measured and used as reference signals.

The test rig configuration is composed of an electric motor, a gearbox and four gear pumps, used for a number of tasks. The four pumps are separated in two groups and each group is mounted on the same shaft. The four pumps have the same number of teeth and rotate with the same speed, taking the motion from the main shaft through a level of gears. As a result the four pumps share also the same characteristic meshing frequency. The only difference of the pumps is their volume ($V_D<V_A<V_C<V_B$ cc).

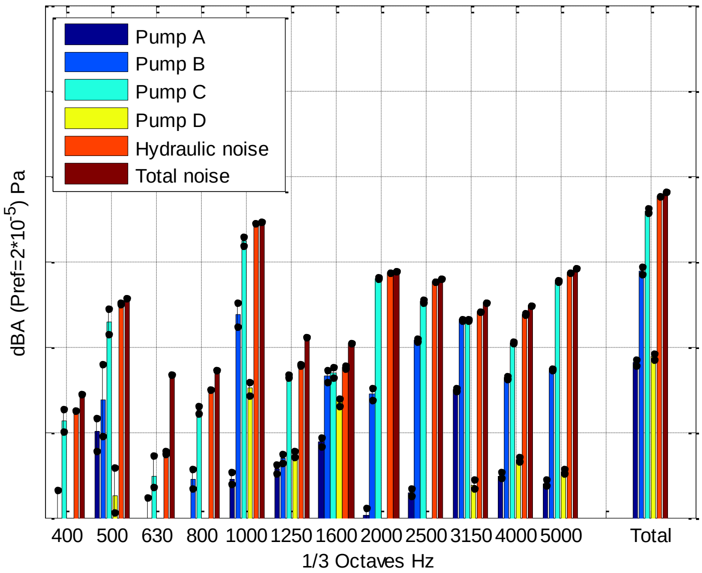

Figure 7: Estimation of the contribution of each pump to the total noise using the G_{con}^{raw} filter.

The measurements were performed using a multi-channel data acquisition system and a number of different sensors, including accelerometers, proximity speed sensors, dynamic pressure sensors and microphones. The sampling frequency was selected equal to 40960 Hz.

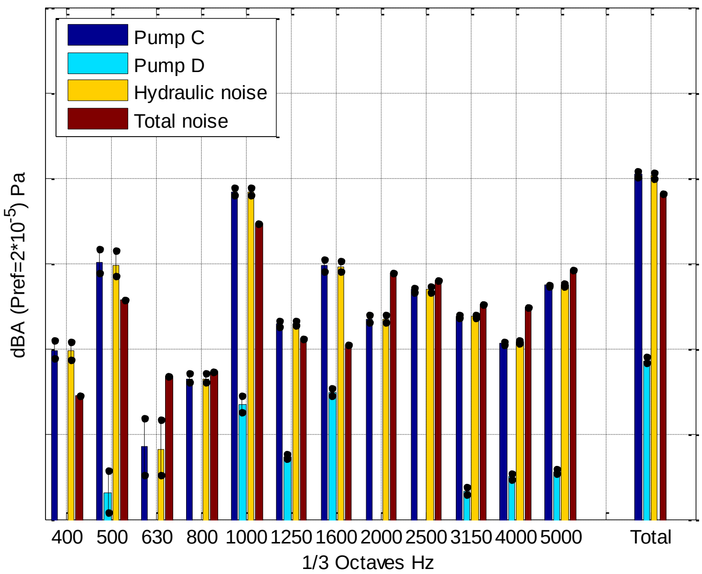

Figure 8: Estimation of the contribution of each pump to the total noise using the G^{res} filter.

Under the predefined conditions it is expected that the measured acoustic signal consists only of the contributions of the four pumps and the "mechanical" noise. Four dynamic pressure signals (one from each pump) were used as reference signals. The acoustic and the pressure signals were firstly resampled in the angular domain using the measured rotation speed. The signals after the resampling are forced to be cyclostationary and have the same number of samples per cycle. All cycles are perfectly synchronized and present the same statistical characteristics. The auto power and the cross power spectral densities of the references and the acoustic signal are estimated. A Hanning window is also used. Moreover the synchronous averaging is performed over all cycles based on the rotation speed of the pumps, leading to the calculation of the periodic part and the residual part of the signals. After the synchronous averaging and taking into account that there are no other mechanical parts rotating at the pumps' speed, the processed signals comprise mainly the noise which is emitted by the pumps.

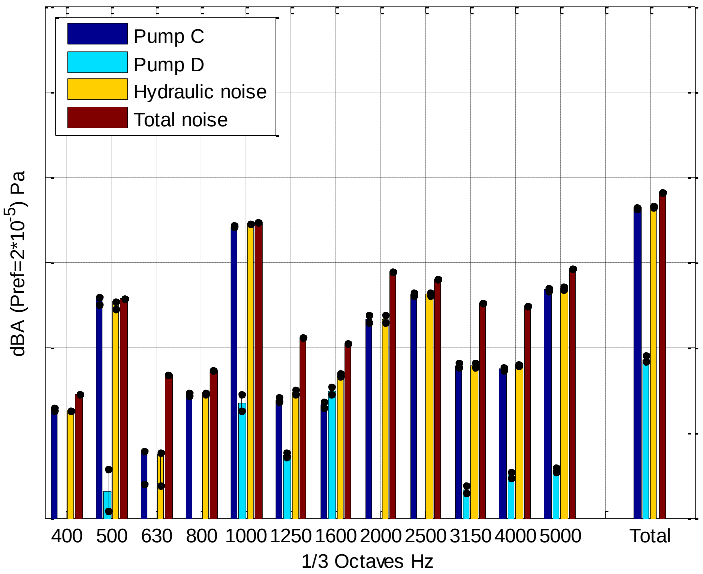

Figure 9: Estimation of the contribution of each pump to the total noise using the G_{con}^{res} filter.

Two cases were further investigated. In the first one it is assumed that there are exactly four sources and that four reference signals are available, one for each source. The two predefined filters G^{raw} and G_{con}^{raw} are estimated (using the raw signals). The filters were further applied on the raw reference signals and the contribution of each pump to the total measured noise is estimated. Moreover the total hydraulic noise and the total measured noise are calculated. Furthermore by using the technique of bootstrapping, the statistical confidence intervals of each of the contributions are estimated. The "A" frequency weighting is used and the results as expressed in dB(A) for 12 1/3 octave bands (400 – 5000 Hz) are presented in Figure 7. The standard reference sound pressure level of 20 micropascals has been used. The contribution of each pump is presented sequentially in bars for each 1/3 octave (bars 1-4). Moreover the estimation of the total hydraulic noise and the measured noise are presented in the same figure (bars 5-6). On the right side of the figure the total sum for each bar over the 17 1/3 octave bands (125 – 5000 Hz) are presented. It is clear that there is no overestimation neither of the contribution of each pump nor of the estimated total hydraulic noise. All the figures have the same scale.

The second case investigated is a more general one. In this case it is assumed that there are more sources at the system than the available references. From the four available references signals, only two are used. The filters G_{con}^{raw} and G^{raw} are calculated and the contribution of the corresponding two sources/pumps is estimated. The bootstrapping is also used in order to estimate the confidence intervals.

Firstly the contributions of the pumps C and D are estimated using only theirs references. The filters G^{res} and G_{con}^{res} are estimated and presented in Figure 8 and Figure 9. The filter G^{res} presents an overestimation in a number of 1/3 octaves. More specifically the contribution of the pump C and as a result the estimated hydraulic noise exceeds the measured noise level. On the other hand the application of the new filter corrects all the overestimations. Moreover by comparing the contributions of the pumps C and D calculated using 4 references (Figure 7) and 2 references (Figure 9) it can be concluded that the new filter estimates with a sufficient accuracy the contribution of the pumps even when the number of references is smaller than the number of sources. Afterwards the contribution of the pumps A and D are estimated by the application of the new filter which was calculated using only the 2 references and firstly the raw signals and secondly the residuals signals. The results are presented in Figure 10 and Figure 11 and are compared with the Figure 7. It is clear that the 2 sources are estimated very well by the new filter calculated on the residual signals (as is imposed by the proposed strategy). Respectively the results obtained by the new filter calculated on the raw signals present significant errors.

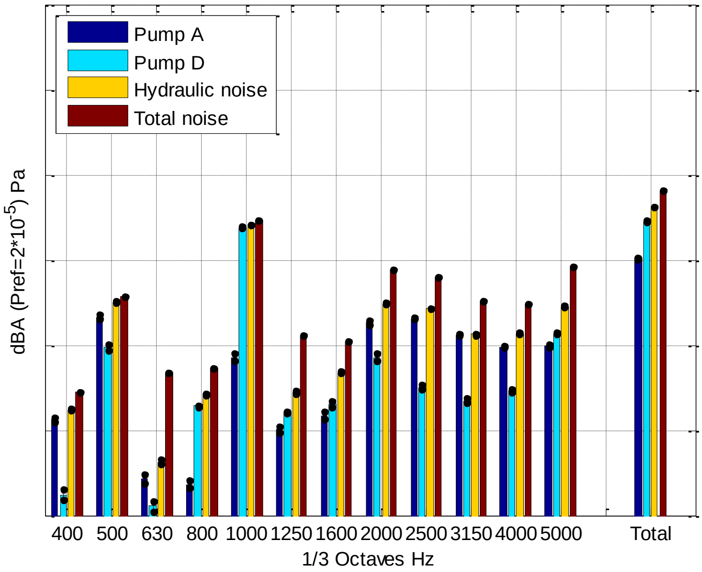

Figure 10: Estimation of the contribution of each pump to the total noise using the G_{con}^{raw} filter.

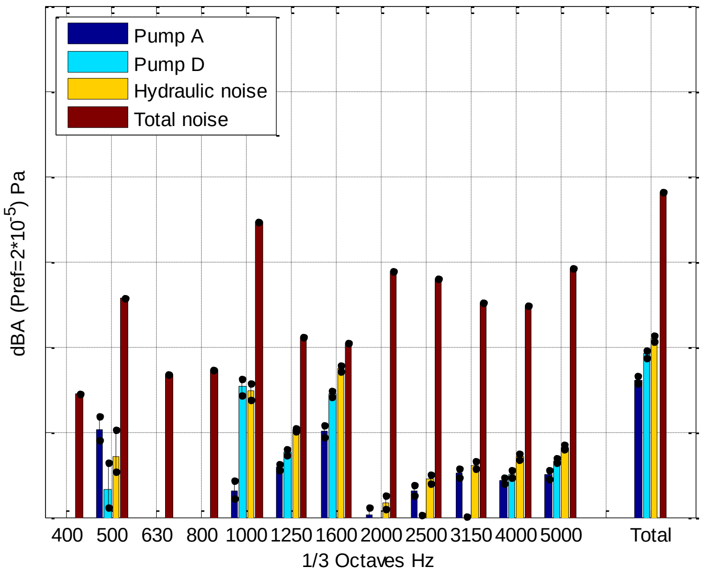

Figure 11: Estimation of the contribution of each pump to the total noise using the G_{con}^{res} filter.

7 Conclusion

In this study a novel source separation approach has been presented, based on cyclic Wiener filtering. A new constrained estimator of the Wiener filter is introduced and a strategy is described in order to help the choice of the part of the signals which should be used for the estimation of the filter. The estimated filters have been applied on acoustic signals captured on an industrial test rig and the results are very promising.

The contribution of the hydraulic noise of each pump has been well estimated even in the worst case where fewer references than sources are available.

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