Structural changes detection with use of operational spatial filter

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Abstract

In the paper authors present a new formulation of spatial filter that enables a filtration of Operational Deflection Shapes instead of mode shapes. The idea is similar to the classical modal filter formulation. In order to find spatial filter coefficients one needs to seek for the vector that will be orthogonal to all ODSs except the one to with the filter suppose to be tuned. Moreover, in this approach one can filter out not only the peaks related to the natural vibrations but also the ones resulting from the excitation. The full paper presents the mathematical description of the spatial filter formulation, its simulation verification and the results of conducted experiments on the laboratory stand. During the experiment the steel frame was excited by the attached electrical motor with unbalance. The structural change introduced to simulate damage had a form of added mass. The estimated spatial filter filtered both peaks resulted from natural vibrations and from excitation.

1 Introduction

In recent time, a few teams of scientists have developed the application of modal filters for the damage detection, and have used the results of such analyses in the Structural Health Monitoring (SHM) [4, 5, 6, 7, 8]. This approach belongs to the group of so called vibration based methods. These have many advantages that has been discussed and verified in various publications [1, 2, 3]. But they have also a few drawbacks: negative influence of the environmental conditions changes, necessity of qualified personnel interaction etc. Almost all of them can be overcome by the modal filter application in the way described in [8]. The modal filter extracts the modal coordinates of each individual mode from a system's output. This is achieved by mapping the response vector from the physical space - the network of measurement points - to the modal space. It was first introduced by Baruh and Meirovitch in 1982 [9] with the aim of overcoming the spill-over problem in control. Generally speaking, it transforms a set of characteristics related with measuring points into one characteristic related to mode shape to which filter is tuned. This output characteristic has only one peak in the natural frequency of the considered mode shape. Mathematical description of the modal filter coefficients identification can be found in [11]. Soon it appeared that it is a very convenient tool and it was applied to: active vibration reduction, the correlation of experimental modal models with theoretical models obtained by Finite Element Method (FEM), the identification of operational forces and finally to damage detection. The paper focuses on the latter group of applications. Among all the advantages, the main weak point of this approach is its limited applicability to output-only data. In the literature, the spatial filtration of the Power Spectral Densities - PSDs (instead of Frequency Response Functions - FRFs) has been described [7]. But this approach very often does not provide the proper results, especially in the rotating machinery, where the harmonic components dominate in the output PSDs. The attempt to solve this problem, has been shown in the paper [10], but solution presented there, requires a personnel interaction and due to this fact its application in SHM is limited. In this paper authors propose a new formulation of spatial filter that enables a filtration of ODSs in place of mode shapes. The idea is similar to the classical modal filter formulation. In order to find spatial filter coefficients one needs to look for the vector that will be orthogonal to all ODSs except the one to which the filter will be tuned to.

2 Theoretical background

At first, it should be reminded what an ODS is. In [12] one can find the following definition: “ODS has been defined as the deflection of a structure at a particular frequency”. Different types of data can be acquired from a measurement, both in time and frequency domain for ODSs determination. The authors focused on frequency domain data and the possible selection included [12]: linear spectra, auto power spectra, cross power spectra, FRFs and ODS FRFs.

The formulation of the spatial filter was described in [13]. In a classical approach, construction of the modal filter requires finding a single vector $\{\Psi_r\}$ that will be orthogonal to all modal vectors $\{\phi_k\}$ but *r*-th - $\{\phi_r\}$, thus the filtration (performed by calculating dot product between the filtering vector and data) will cancel out the contribution of all modes except the *r*-th, providing a function of a single mode only. Important difference between modes and ODSs is that modes are strictly related to natural frequency of the structure whereas ODSs are defined for any frequency, stating a problem of selection of particular vectors that are suitable for filter construction. In the presented method, similarly to the classic technique mentioned above, only these ODSs were taken into account, which correspond to the natural frequencies and possibly rotational velocity harmonics of an object and they will be selected by the method of peak picking. In real life application, excitation forces will strongly influence system responses, therefore presenting a challenge in proper ODSs selection, but in this particular case this matter seems to pretty straightforward.

Mathematical criteria needed to be met in order to find a proper filtering vector $\{\Psi_r\}$ tuned to extract only contribution of mode corresponding to *r*-th ODS is presented by Eq. 1 [11]. However, in this case $\{\phi_k\}$ does not denote *k*-th modal vector but *k*-th ODS vector. Assuming that none of the following vector is zero length, $\{\Psi_r\}$ must be orthogonal to all ODS vectors but $\{\phi_r\}$.

$$\{\phi_k\}^T\{\Psi_r\} = \begin{cases} 1, & r = k \\ 0, & r \neq k \end{cases} \tag{1}$$

Equation (1) can be expanded into a set of equations (2), and then be solved with respect to $\{\Psi_r\}$ in order to find a proper filtering vector.

$$\begin{bmatrix} \{\phi_1\}^T \\ \vdots \\ \{\phi_n\}^T \end{bmatrix} \{\Psi_r\} = \{\rho_r\}, \quad \text{where} \quad \{\rho_r\}^T = \{a_1 \dots a_k\}, \quad a_i = \begin{cases} 1, & r = k \\ 0, & r \neq k \end{cases} \tag{2}$$

At this point it should be stated that the number of equations and unknowns determine solvability of the system. If there are less equations than unknowns, that is, less responses measured than ODSs taken, it is impossible to find a vector that would be orthogonal to all but one ODS. Such a problem can be overcome by limiting band of a spectra taken into consideration to a one, which has just as many peaks as response points. This approach seems to be relevant especially in real life application where physical structures have infinite number of DOFs and there is a limited number of sensors. In the case presented in this paper ODS matrix is square and invertible (providing that ODS vectors are linearly independent), therefore only one exact solution exists and can be found by solving equation (2).

The spatial filtration is done by multiplication of vector $\{\Psi_r\}^T$ by the response spectra matrix, as stated in the equation (3).

$$\eta(\omega) = \{\Psi_r\}^T \begin{bmatrix} \{p_1(\omega)\}^T \\ \vdots \\ \{p_n(\omega)\}^T \end{bmatrix} \tag{3}$$

where $\eta(\omega)$ denotes ODS filter output and $p_i(\omega)$ is auto power spectra of *i*-th response point.

3 Simulation verification

Simulation verification of the described spatial filter was shown in [13]. Here only some abbreviation are presented for completeness of the article. For this purpose a model of linear, time-invariant, mass-damper-spring mechanical system was defined, its scheme is showed in Figure 1.

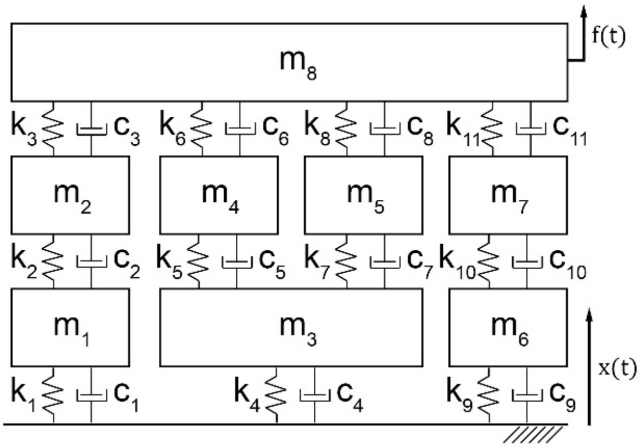

Figure 1: Model used for simulation

The model consists of eight masses interconnected with each other using spring (proportional stiffness) and damper (proportional damping) elements. The output of the system are the displacements of all masses. Impulse type of excitation force was applied to mass number 8. Selection of wide-band input signal reassures that the object under scope is well excited and contribution of all modes will be seen in the output of the system. In the simulation zero initial conditions were specified, exponential window was applied to the output data. The eigenvalue problem for this system was solved, modal parameters are gathered in Table 1.

Mode no.	Natural frequency [Hz]	Damping coefficient [%]
1	11.87	0.356
2	20.38	0.917
3	24.15	1.404
4	26.45	1.416
5	36.17	1.702
6	44.26	2.425
7	55.86	2.515
8	65.47	3.254

Table 1: Dynamic properties of examined model

In this particular case ODS created from Auto Power Spectra (APS) responses was chosen for the construction of a spatial filter. Welch estimator was utilized to obtain power spectra density of each output from the time displacement vector of each mass. Results of the estimation of APS of the responses are presented in Figures 3 in logarithmic scale.

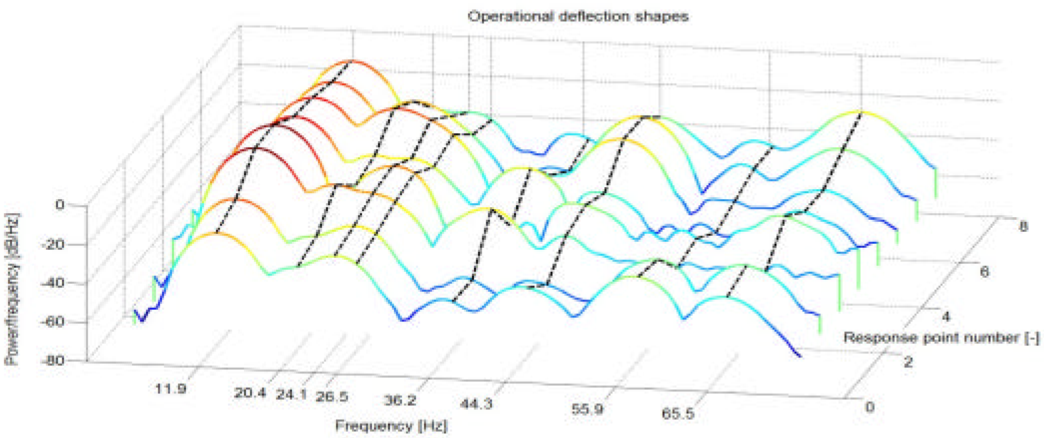

Figure 2: PSDs of the system output

Thick black lines mark the ODS vectors at natural frequencies of the structure. These vectors will create an ODS matrix that will be used for the construction of the spatial filter. Number of the degrees of freedom of the system is equal to the number of its modes or natural frequencies, therefore the count of ODS vectors chosen will match the size of each vector, forming a square ODS matrix.

For the model presented in the beginning of the article, eight filters, tuned to ODS at each natural frequency of the system, where constructed and applied. The output of the filter is illustrated in Figure 4.
In the output of the filters number 1, 2, 5, 6, 7, 8 a single peak at the frequency to which the filter was tuned is visible. Some minor residual traces of can be spotted in the remaining frequency bands, however their amplitude does not affect the general image. Isolated single ODS can be easily traced revealing changes of in the structure, such as damage occurrence. Output of two remaining filters, 3 and 4, is quite distorted and brings little value for a potential SHM system.

Further investigation of results in order to explain such a low quality of filtration, for these two vectors, showed, that the factor that has influenced the filter is a weak linear independence between vectors 3 and 4 which can be spotted in Figure 4 – these ODS vectors are almost parallel. As a result filters 3 and 4 cannot effectively cancel out the contribution of modes 4 and 3, respectively. Operational deflection shapes $\{\phi_{3,4}\}$ to which filters were tuned and their filtering vectors $\{\Psi_{3,4}\}$ are nearly orthogonal providing a low dot product, which then has to be compensated by the length of filtering vector as the result must be equal to 1. Increased length amplifies the residues of other modes, making the outcome noisy.

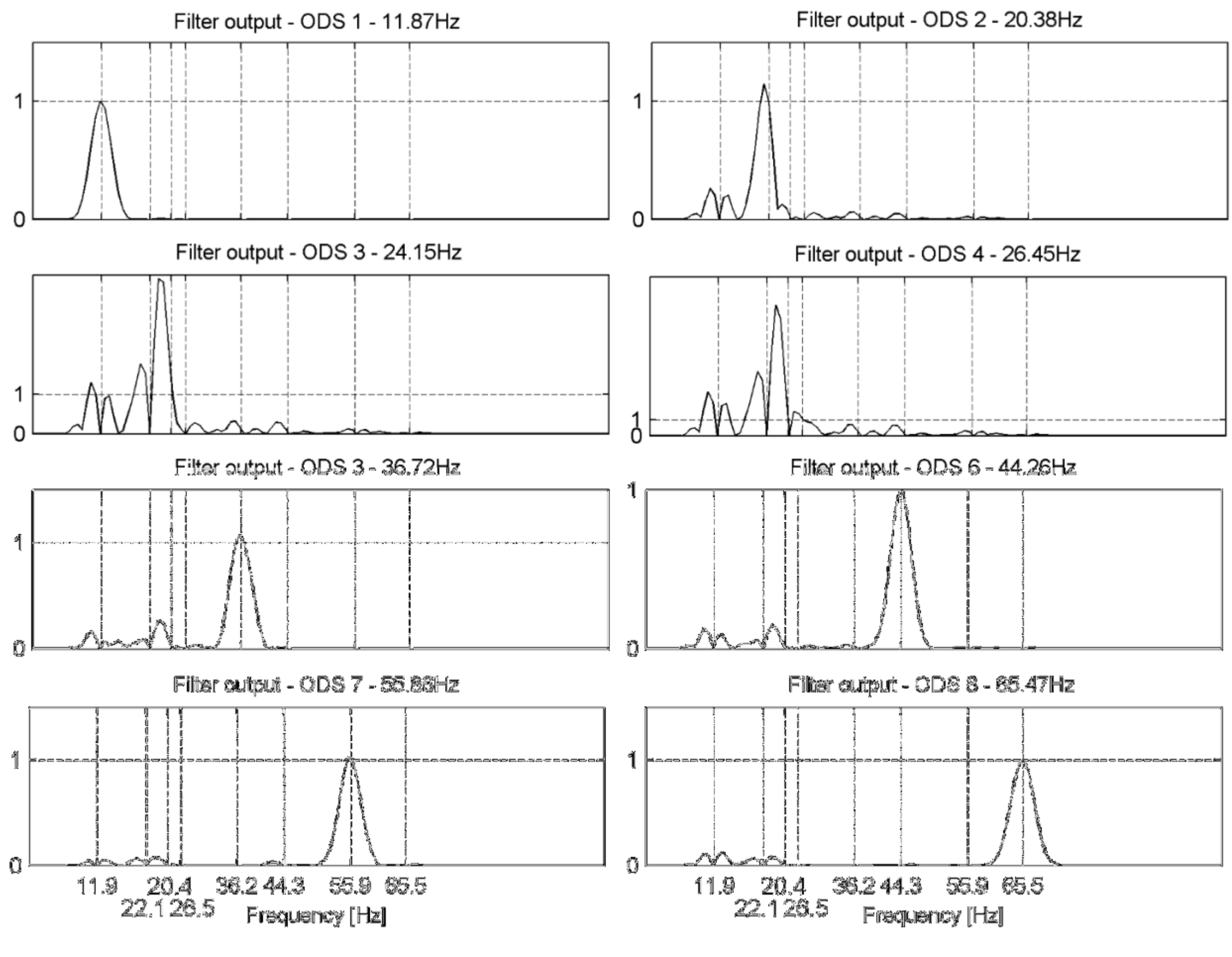

Figure 4: Results of filtration. Each graph presents and output of spatial filter tuned for ODS corresponding to natural frequency of the system

Comparison of filtration results obtained for classical modal filter and this new approach was presented in [13], and proved its good efficiency.

4 Experimental verification

To test ability of the spatial filter to the structural damage detection the laboratory test was performed. The stand used for experimental validation of the proposed structural health monitoring procedure consists of a steel - aluminium frame excited with a modal hammer. Additional harmonic excitation was applied by electrodynamic shaker. This excitation was introduced to test the method ability of harmonic components filtration its frequency amounted 150 Hz. Vibrations were measured by accelerometers placed on the frame. A photo of the test setup with sensors and measuring equipment is presented in Figure 5, and the network of measuring points is presented in Figure 6. The frame has been tested for different structural changes. This change had a form of added mass. Total mass of the upper aluminium beam amounted 0.544 kg, added masses amounted 25 g and 50 g. It represents 5 % and 10 % of the beam mass. During the tests, the FRFs of the object in the form of inertance were recorded. The frequency range of measurements was chosen as 0 – 512 Hz with the resolution 0.25 Hz. The FRFs were averaged 7 times in the frequency domain for random error compensation.

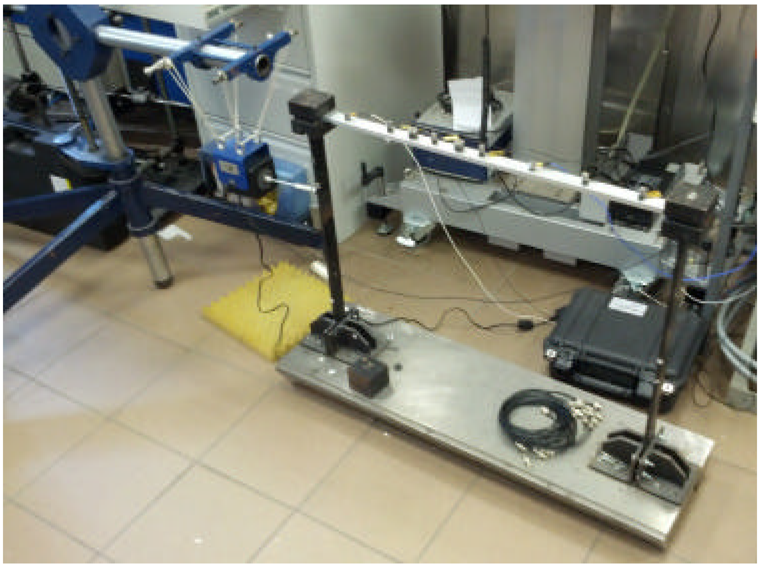

Figure 5: Photo of the test setup

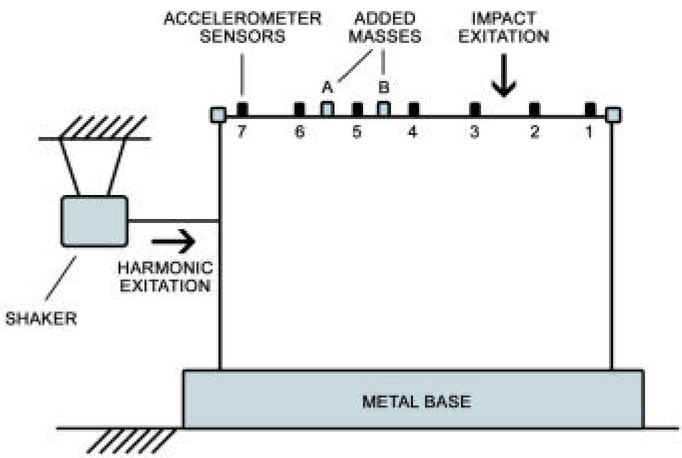

Figure 6: Network of measuring points

In the selected frequency range there are three natural frequencies of the stand, and for these frequencies the ODSs were obtained from the response PSDs. Additionally the harmonic component at 150 Hz was selected. In Figure 7 the PSDs of the frame with dashed lines representing selected ODSs are presented.

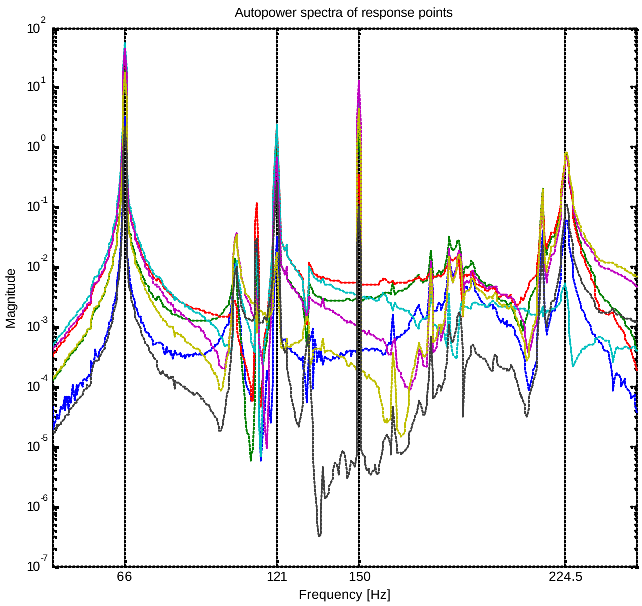

Figure 7: PSDs of the reference frame output

For this 4 ODSs appropriate spatial filters were identified, and used for spatial filtration of the output characteristics. Next the consecutive tests were performed with added masses in two stages 25 g and 50 g. Data obtained from these tests were also filtered with use of the spatial filters identified for the reference frame. The results of structural changes detection with use of such a procedure are presented in the figures 8 and 9. Finally the total damage index defined by Formula (4) was calculated for every test. It values are shown in Figure 10.

$$DI = \frac{\int_{w_s}^{w_f} \left| x_i(w) - x_{ref}(w) \right|^2 dw}{\int_{w_s}^{w_f} x_{ref}(w)^2 dw} \tag{4}$$

where: w_s, w_f – starting and closing frequency of the analyzed band,
x_i, x_{ref} – characteristic in the current and reference state respectively.

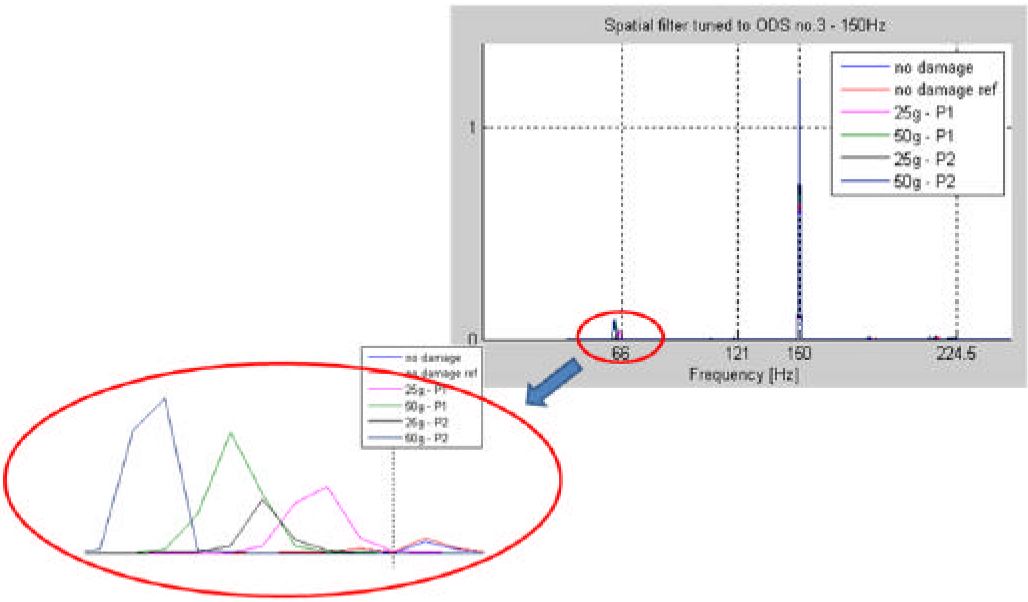

Figure 8: Output of spatial filter tuned to ODS no. 3

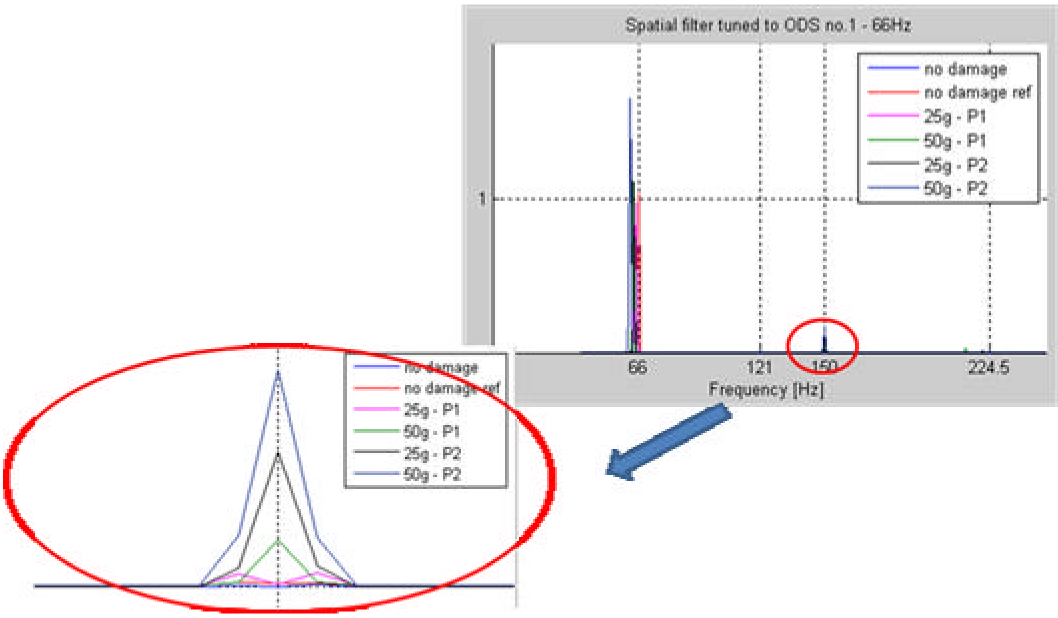

Figure 9: Output of spatial filter tuned to ODS no. 1

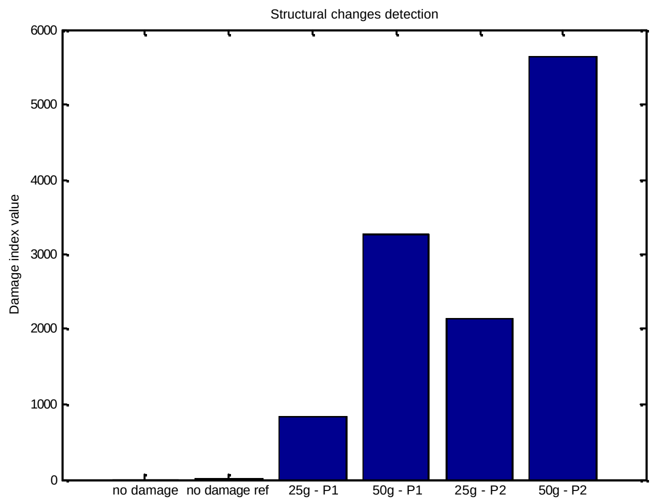

Figure 10: Damage index values for performed tests

Presented results shows that the spatial filter described in the paper works properly for the measured data and its useful in structural changes detection. Also the harmonic component can be filtered with use of this approach, and this makes the damage detection even easier, because the unfiltered peak due to structural change will not change its frequency.

5 Final conclusions

The paper presents a new method for construction of a spatial filter with the use of output only data that is able to filter ODSs. The fact that operational data is directly used, reduces requirements for computational power as well as decreases complexity of prospective SHM system, giving a great deal of potential to this method. The presented numerical and experimental verification showed that, with use of in-operational data it is possible to construct the spatial filter with comparable results to the classical modal one. Application of proposed spatial filter in structural changes detection showed proper results. Another conclusion is that with use of this filter it is possible to deal with harmonic component.

Acknowledgments

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