On the time synchronous average in planetary gearboxes

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Abstract

Generally, in planetary gearbox diagnostics, transducers are placed on the gearbox case near the ring gear. The relative angular position of the planet gears with respect to the transducer is a pivotal information for the evaluation of vibration signals related to planet/sun gears. This angular position is usually unknown, or it is known with a large tolerance. The present work seeks to propose two alternative methods for the identification of the angular position of the planet gears with respect to the transducer. The first one is based on the study of how the power flows inside the Time Synchronous average of the ring gear, whilst the second method is based on a modified statistical parameter such as the Crest Factor. The effectiveness of these methods is assessed on the basis of actual vibration signals acquired from a faulty planetary gearbox.

1 Introduction

Gearbox diagnostic algorithms use vibration data collected from accelerometers located on the gearbox housing. These vibration signals tend to be an hodgepodge of vibrations associated with all of the components within the gearbox. To provide an effective fault diagnosis, it is mandatory to understand which component is causing a given vibration pattern.

Time synchronous averaging (TSA) has been shown to be a useful tool for extracting gear mesh vibrations from composite vibration signals since it enables the extraction of periodic signals from noise-polluted signals [1–3]. The resulting vibration signal corresponds to one compete revolution of the gear under consideration, and thus changes in the vibration waveform due to damage on individual teeth can be identified.

The application of the TSA for the extraction of periodic waveforms in ordinary gearbox was proposed by Braun in the mid 70s [1]. However, such TSA techniques relate to gears with fixed vibration transfer paths from the source of the vibration to the transducer. With planetary gears the vibration transfer path is not fixed but it is subjected to variation.

A technique for the evaluation of the TSA vibration signals associated with the sun and planet gears of a planetary gearbox was proposed by P.D. McFadden in the early 90s [4]. This work demonstrates that, for planetary gearboxes with certain geometric properties, the averaged vibration signals can be extracted from a vibration signal captured by a single fixed-frame transducer. Subsequent studies validated this research and presented slight variations on the technique [5, 6]. However, the fundamental methodology remains unchanged.

The TSA of the planet/sun gears can be obtained if and only if, the relative position of the planet gears with respect to a transducer placed on the ring gear is known a priori. In [8] McFadden suggested that the position of each of the planet gears with respect to the transducer, could be estimated directly form the TSA of the ring gear by identifying the locations of the maximum vibration amplitude. This identification could be more effective from the amplitude modulated signal of the ring gear. For small planetary gearbox, this approach could be not effective and the planet position cannot be identified leading to a poor evaluation of the planet gear TSA, as outlined in the following.

The aim of this work is to propose two alternative methods for the identification of the position of each of the planet gears with respect to the transducer.

The paper is organised as follows. After a brief introduction and problem statement given in this Section, the TSA algorithm proposed by McFadden is outlined in the next one. After that, the two proposed methods for the evaluation of the planet gear positions are presented in Section 3. Finally, the effectiveness of the two methods is discussed on the basis of real data in Section 4.

2 TSA in planetary gearbox

McFadden and Smith [7] show that as a given planet gear approaches the transducer, the measured vibration level increases, whilst as the planet gear moves away from the transducer the measured vibration level decreases. The transfer function between the transducer and the planet gear is $h(t)$ and has a period of one carrier revolution T_c, Figure 1. Thus, planet signal $x(t)$ as seen by transducer j is given by $h_j(t)x(t)$.

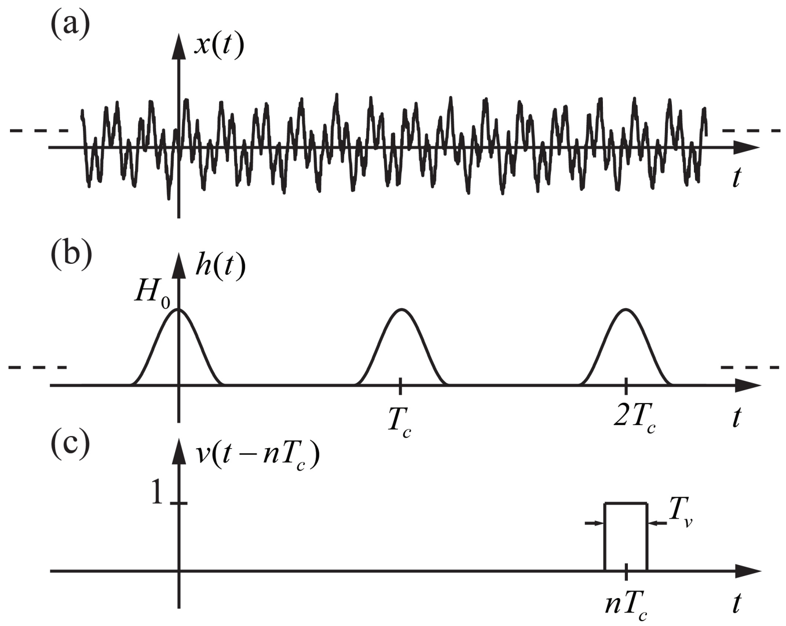

Figure 1: Planet vibration signal measured by a fixed transducer [4]: (a) Time planet vibration signal, (b) Transfer function between planet and transducer (c) windowing function $v(t-nT_c)$, where n is an integer number

In order to extract the planet/sun signal McFadden stated that: *"when a given planet gear is near a transducer, the vibration measured by the transducer are dominated by the meshing of that specific planet gear with the sun and ring gears."* Thus, during each passing of a given planet, a small data window can be collected. It can be assumed that over the width of such a window, the transfer function between the accelerometer and the region of tooth contact will remain constant. The planet gear teeth in mesh can be determined at each carrier revolution, and the window of data can be stored in a buffer according to the meshing tooth. This process is than repeated several times, in order to obtain a window of data for each tooth on the planet gear. The so arranged buffer includes the vibration signal for a complete revolution of the planet gear. Several buffers can then be averaged in order to obtain the TSA signal of the gear of interest, Figure 2.

Mathematically speaking, let define a windowing function centered at time $t = nT_c$, where n is an integer number. The time at which $h_j(t)$ reaches its maximum is defined by $v(t-nT_c)$, Figure 1 (c). The subsequent windowed vibration signal is given by expression $h_j(t)x(t)v(t-nT_c)$.

Let the window width be chosen as an integer number of tooth mesh periods T_m, given by [8]:

$$T_v = N_v T_m \tag{1}$$

If N_v is chosen to be appropriately small, then the amplitude of $h_j(t)$ can be assumed to be a constant $H_{j,0}$ over the entire window, and the vibration signal becomes:

$$h_j(t)x(t)v(t-nT_c) = H_{j,0}x(t)v(t-nT_c) \tag{2}$$

Once a window of vibration data has been obtained, it must be mapped into the appropriate location in a buffer for synthesizing the planet/sun gear vibration signal. To determine this location a sampling function $g(t) = g(t-nT_g)$ can be used, where T_g is the rotation period of the gear of interest (planet/sun). The convolution operator can be used, leading to $[H_{j,0}x(t)v(t-nT_c)] * g(t)$, in order to map the windows into the appropriate location. If the tooth number of the gear of interest is N_g, once N_g windows have been mapped, all of the teeth of the gear under consideration will be captured.

In order to extract the TSA vibration signal from the measured one, a large number (N_e) of synthesized signals (buffers) must be captured and averaged. The TSA of the gear of interest is given by:

$$x_g(t) = \frac{1}{N_e N_v} \sum_{n=0}^{N_e N_g - 1} [H_{j,0} x(t) v(t - nT_c)] * g(t) \tag{3}$$

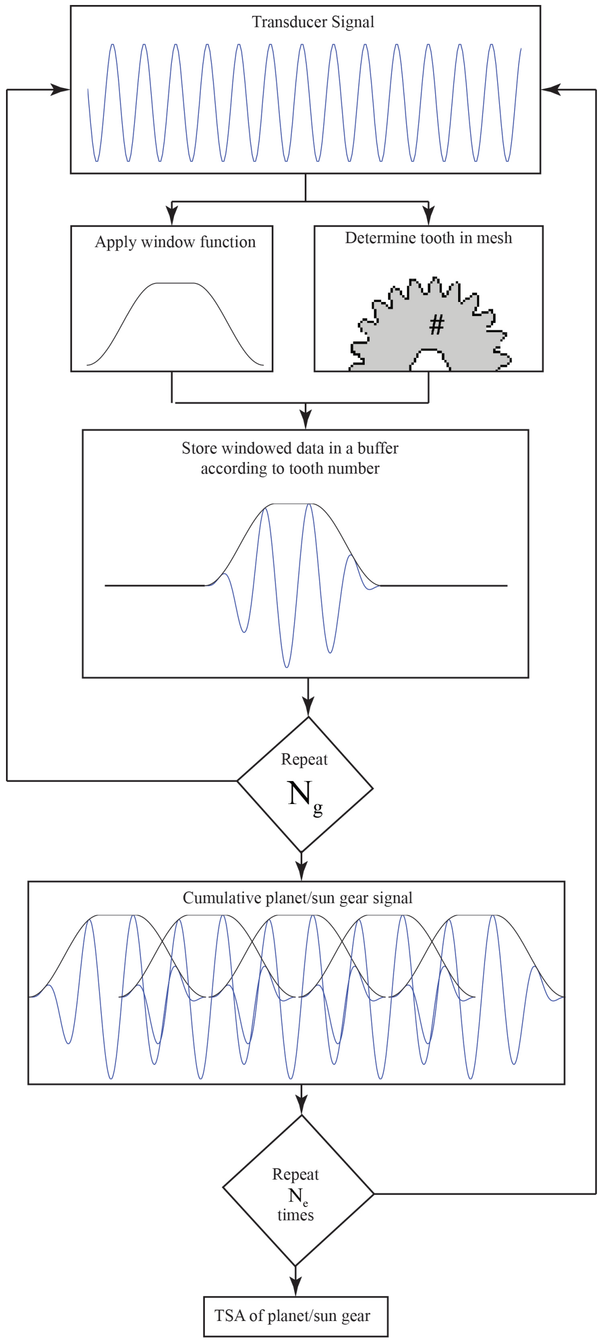

Figure 2: TSA block diagram for planet/sun gear. N_g is the tooth number of the gear of interest. N_e is the number of averages.

3 On the planet-transducer position

Function $h(t)$ is directly related to the mass, damping and stiffness properties of the gearbox. In some cases the combination of these properties leads function $h(t)$ to be particularly flat. Thus, no amplitude modulation effects can be visible in the ring gear TSA, also after the application of the amplitude demodulation techniques. The planet gear position relative to the transducer cannot be pointed out, leading to a poor evaluation of the planet gear TSA.

3.1 Method A: Power flow

The relative position of each of the planet gears with respect to the transducer could be estimated by studying how the power flows in the vibration signal. In particular, the planet position could be determined from the power flow of the ring gear TSA. As a matter of fact, each time a planet passes under the transducer, there is an increase of the power released within the signal.

The operator that describes how the power flows within the signal was introduced by J. Antoni few years ago [9]. Let $x(t)$ be a continuous time signal, the mean instantaneous power is defined as:

$$P_x(t) = \sum_{\alpha \in \mathscr{A}} P_x^{\alpha} e^{j2\pi\alpha t} \tag{4}$$

where P_x^{α} is the cyclic powers of the signal at cyclic frequencies α, defined as:

$$P_x^{\alpha} = \lim_{T\to\infty} \frac{1}{T} \int_T |x(t)|^2 e^{-j2\pi\alpha t} dt \tag{5}$$

The set $\mathscr{A}$ in equation (4) embraces all the cyclic frequencies inside $x(t)$.

In order to obtain the position of the planet gears with respect to the transducer, only the cyclic frequency corresponding to the number of planets of the planetary gearbox is taken into account. In particular, the whole procedure could be summarised as follows:

- ✓ Evaluation of the ring gear TSA
- ✓ Evaluation of the P_x^{α} of the ring gear TSA
- ✓ Reconstruction of the P_x, including in the set $\mathscr{A}$ only the cyclic order equal to the number of planet gears
- ✓ The first maximum in the P_x function gives the phase of one planet gear with respect to the transducer.

Once one planet gear phase is evaluated, the phases of the other planet gears can be obtained by taking into account the angle between two consecutive planet gears, or by taking into account the other maxima.

3.2 Method B: Statistical Parameter

On the other hand, the position of a particular planet gear with respect to the transducer can also be determined by the analysis of a simple statistical parameter. A counterpart of the function P_x can be obtained by a tooth-wide evaluation of a “modified”Crest Factor (MCF). The MCF is defined as:

$$MCF = \frac{x_{peak-peak}(t)}{RMS(x(t))} \tag{6}$$

where $x_{peak-peak}(t)$ is the peak to peak value and $RMS(x(t))$ indicates the RMS value. Generally speaking, the Crest Factor is a measure of how extreme the peaks are in a waveform compared to the mean value. The peak to peak value is used in equation (6) in order to increase the sensitivity of normal Crest Factor. The MCF evaluated by equation (6) is not a function of time, but a single numerical value. In order to obtain a function which could be related to the signal power flow, equation (6) is evaluated on a signal portion embracing a single ring gear tooth. Finally, the tooth-wide MCF is filtered around the order of the planet carrier rotation which corresponds to the planet gear number. The maximum value of the so processed MCF gives the position of one planet gear with respect to the transducer. The whole procedure could be summarised as follows:

- ✓ Evaluation of the ring gear TSA
- ✓ Evaluation of the MCF for each ring tooth
- ✓ Filtering the tooth-wide MCF function around the order of the planet carrier rotation which corresponds to the planet gear number
- ✓ The first maximum in the filtered tooth-wide MCF function gives the phase of a planet gear with respect to the transducer.

As previously stated, once one planet gear phase is evaluated, the phases of the other planet gears can be obtained by tacking into account the angle between two consecutive planet gears, or by taking into account the other maxima.

4 Experimental data analysis and discussion

This section aims to test the effectiveness of the proposed methods for the evaluation of the planet gear positions with respect to the transducer on the basis of experimental data. Tests were performed in a test-rig designed and built up at the Engineering Department of the University of Ferrara. The test-rig consists of a base, including two induction motors controlled by inverters and a planetary gear unit (Figure 3). In more detail, the driving induction motor (BN80C2) is controlled in a feedback speed loop; its speed is evaluated by an encoder with 360 pulses per revolution. The load induction motor (BN132MB4) is controlled in a feedback torque loop, while the speed is evaluated by an encoder with 3600 pulses per revolution. Table 1 lists the data of the induction motors.

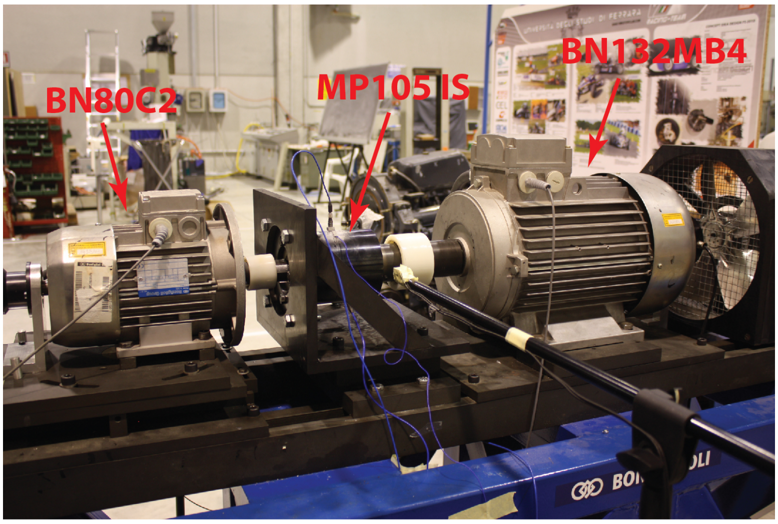

Figure 3: Test-rig

	BN80C2	BN132MB4
Nominal power [kW]	1.5	9.2
Nominal torque [Nm]	5.1	61
Nominal speed [rpm]	2800	1440
Number of poles per phase winding	2	4

Table 1: Induction motor data

The planetary gear unit (MP 105 IS) is a single-stage system containing a sun gear (27 teeth), three planets (39 teeth) and a ring gear (108 teeth) for a global speed reduction ratio of 5. Two localised tooth faults were artificially introduced on one planet gear. Figure 4 depicts the two faults, namely LFP1 and LFP2. In particular,

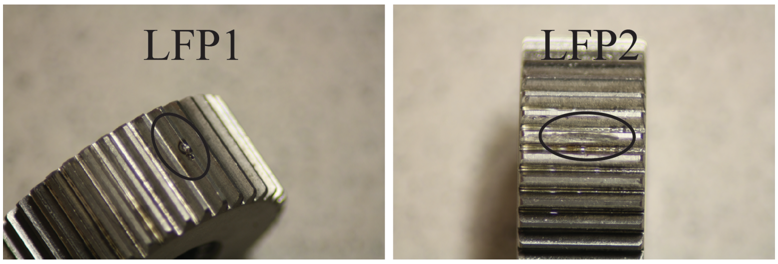

Figure 4: Localised tooth faults

LFP1 is a small tooth fault introduced on the tooth flank with an electric pen drive. The LFP2 fault, which is introduced via a drilling process, embraces approximately half of the tooth flank.

During tests, the vibration signals were acquired by means of a piezoelectric accelerometer (frequency range 1 to 12000Hz) mounted in radial direction on the gearbox case near the ring gear. A one pulse per revolution optical tachometer is mounted on the gearbox output shaft (planet carrier). Signals where acquired to an extent of 60s with a sample frequency of 102kHz. This hight sample frequency was used in order to properly acquire the tachometer signal. Vibration signals were subsequently downsampled at 12kHz during post processing. Tests were performed at different conditions of driving speed and applied torque. The results presented in this work are relative to a nominal driving motor speed of 20Hz and nominal output shaft torque of 12Nm.

Figure 5 depicts the time signals captured from the accelerometer for both sound and faulty conditions. The vibration amplitude related to the two fault conditions is not increased compared to the sound one. In particular, the modulation effect due to the planet gear passage toward the accelerometer is not visible.

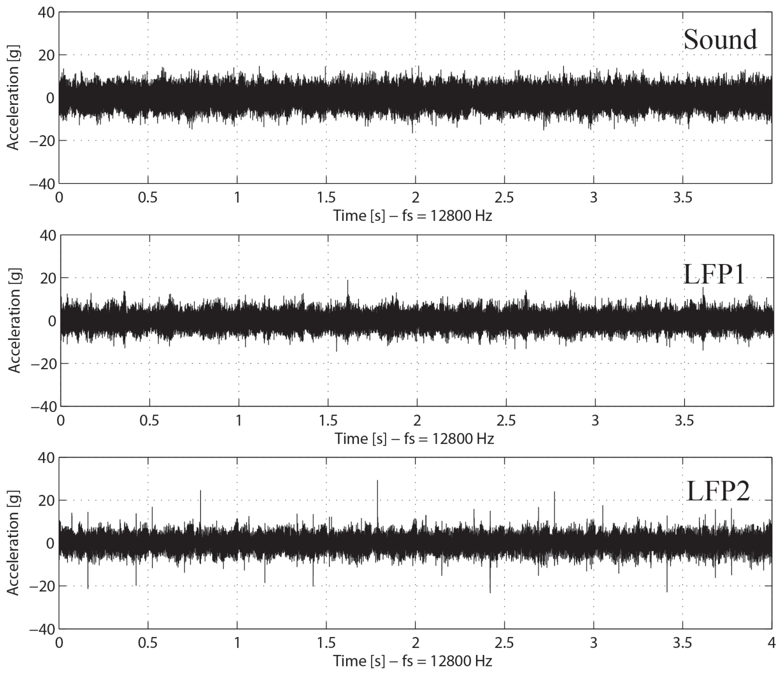

Figure 5: Time signal for 2 revolutions of the planet carrier: Sound, LFP1 fault, LFP2 fault

Diagnostic informations about the faulted planet gear can be obtained by extracting, from the global vibration signal captured by the accelerometer, the signals of each planet gear. In oder to do that the TSA technique proposed by McFadden [4] can be used. The application of this technique relies on the knowledge of the position of the planet gears with respect to the accelerometer. This position can be determined from the ring gear TSA by identifying the locations of the maximum vibration amplitude. Figure 6 depicts the ring gear TSA for both sound and faulted conditions.

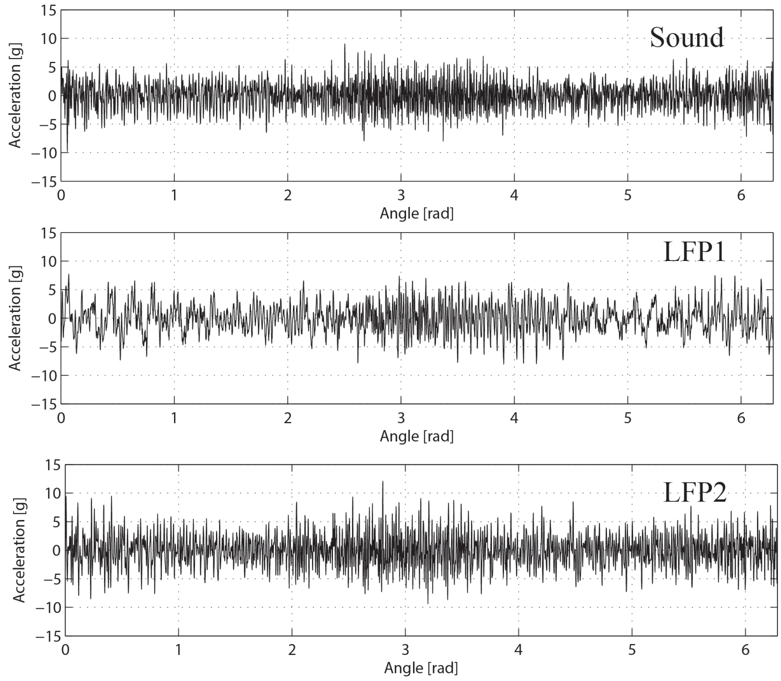

Figure 6: Ring gear TSA: Sound, LFP1 fault, LFP2 fault

The three ring gear TSAs are essentially the same, sure enough these represent the ring gear signal which does not change from the sound to the faulty gearbox conditions. The relative position of each of the planet gears with respect to the transducer is not visible. Because of the geometrical and material properties of the gearbox under test, the transfer function between the transducer and the planet gear (Figure 1 (b)) is flat for the most part. This is mainly due to the mass, damping and stiffness properties of the gearbox. In this case the position of the planet gears is not detectable leading to a poor evaluation of the planet gear TSAs.

Figure 7 depicts the core of this work. In particular Figure 7 shows the results of the application of the two proposed methods on the ring gear TSA for both the LFP1 and LFP2 fault conditions, respectively. The two methods, the first one based on the P_x and the second one based on the MCF gives the same results, i.e. the points related to the maxima and minima of the two functions are the same.

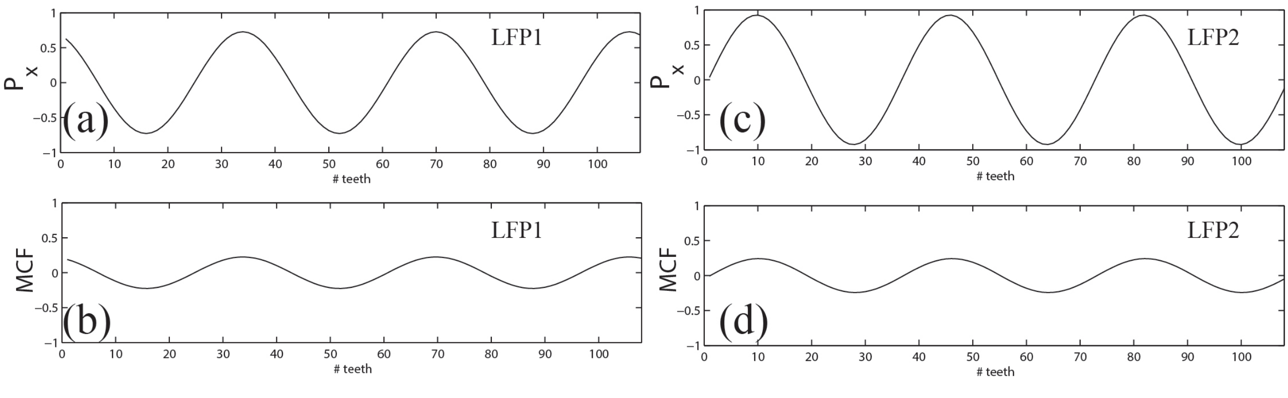

Figure 7: P_x and MCF functions: (a) P_x function for the LFP1 fault, (b) MCF function for the LFP1 fault, (c) P_x function for the LFP2 fault, (d) MCF function for the LFP2 fault

In particular, it is possible to see that for the case of the LFP1 fault (Figure 7 (a) and (b)) the first maximum is reached around ring gear tooth 34. This means that the starting position of one planet gear, which could be the planet gear 1 without loss of generality, is shifted of an angle covered by 34 ring gear teeth with respect to the transducer. By inspecting the other two maxima of the functions, or via geometric considerations, one can conclude that the other two planet gears are shifted by an amount of 70 and 106 ring gear teeth with respect to the transducer (Figure 7 (a) and (b)). Analogous considerations can be performed for the LFP2 fault. Specifically, the first maximum is reached around ring gear tooth 10, which means that one planet gear is shifted of an angle covered by 10 ring gear teeth with respect to the transducer (Figure 7 (c) and (d)). The other two planet gears are shifted of 46 and 82 ring gear teeth respectively.

These position informations can be used in order to extract the vibration signal of the planet gears with the TSA technique proposed by McFadden. Figure 8 depicts the result of this operation for the LFP1 fault case, where the starting position of the averaging process is obtained by shifting the vibration signal of an angle covered by 34 ring gear teeth. The averaging process can extract the vibration signal related with each planet gear in a precise manner. As depicted in Figure 8 (a) a small variation of the vibration amplitude can be seen in the planet gear 1 TSA. This small amplitude variation is not suitable for a sure fault detection and further analyses are needed. However, this does not affect the effectiveness of the proposed methods. In particular, Figure 9 compares the results of the extraction of the planet gear 1 TSA with two different signal shifts. Firstly, the signal is shifted of an angle covered by 34 ring gear teeth (Figure 9 (a)) and finally the signal is shifted of an angle covered by 88 ring gear teeth (Figure 9 (b)).

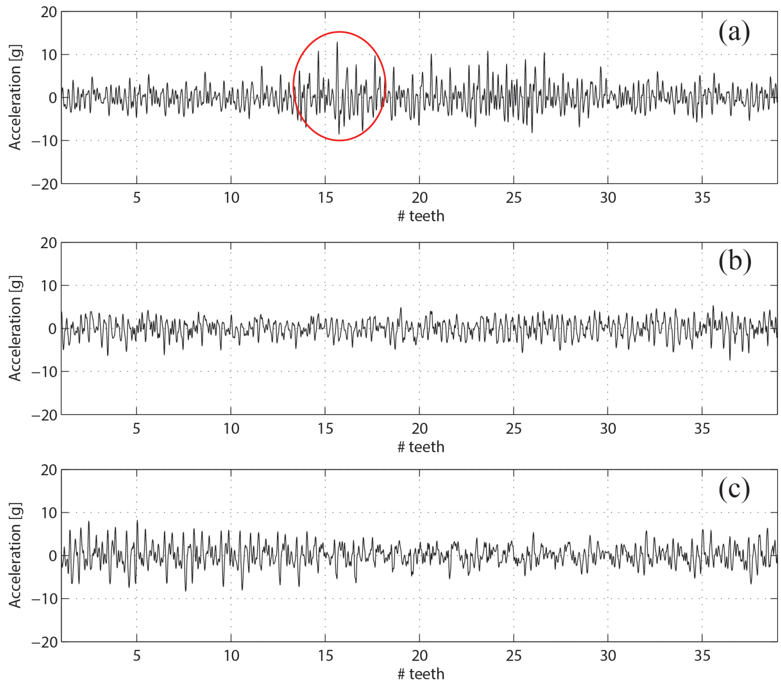

Figure 8: TSA of planet gears for LFP1 fault with an initial phase shift of 10 teeth: (a) Planet gear 1, (b) Planet gear 2, (c) Planet gear 3

The last shift, which corresponds to a minimum of both P_x and MCF functions, is the farthest position of planet gear 1 with respect to the transducer. As one can see, no evidence of variation in the vibration amplitude can be detected in Figure 9 (b). This result highlights the importance of the knowledge of the relative position of the planet gear with respect to the transducer, even in the case on which the transfer function $h(t)$ is particularly flat.

Figure 10 plots the result of the TSA of the planet gears for the LFP2 fault case. These averages are performed by shifting the signal for an amount corresponding to the first maximum of the P_x and MCF functions. It is possible to see a strong variation in the signal amplitude of the planet gear 1 TSA (Figure 10 (a)). Moreover, a small variation in the vibration amplitude can also be detected in the TSA of planet gear 3 (Figure 10 (c)). This phenomenon is due to a combination of causes related to the fault size and to the flat shape of the transfer function $h(t)$. Taking into account the planet-ring gear tooth mesh sequence, one can see that when tooth j of planet gear 1 is meshing, also the tooth $j+8$ of planet gear 3 engages a tooth of the ring gear. Accordingly to that, when tooth 8 of planet gear 1 is meshing, tooth 16 of planet gear 3 is meshing too. Thus, when an impulse is generated during the mesh of the tooth 8 of planet gear 1, a variation in the vibration amplitude of planet gear 3 could also be detected around tooth 16. The latter is not a real fault, but it is a “ghost component”correlated

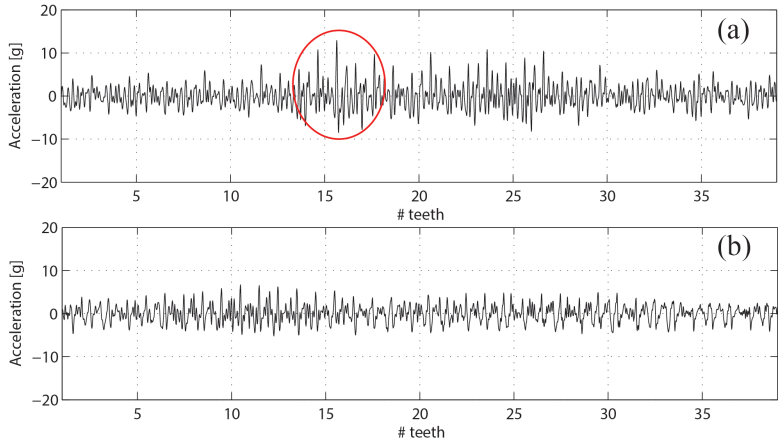

Figure 9: TSA of planet gear 1 for LFP1 fault: (a) initial phase shift of 34 teeth, (b) initial phase shift of 88 teeth

to the flat shape of the transfer function $h(t)$. Because the two impulse responses are correlated, only one planet gear is faulted.

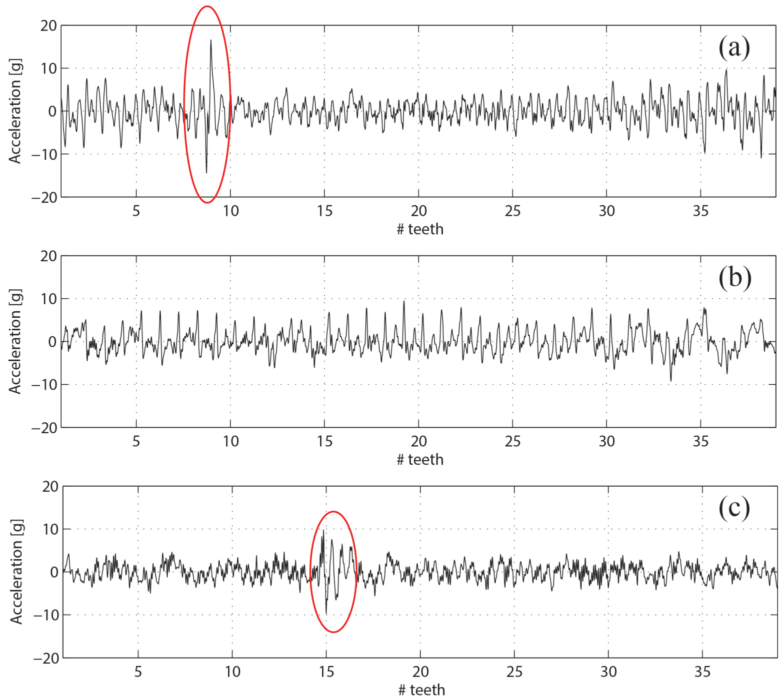

Figure 10: TSA of planet gears for LFP2 fault with an initial phase shift of 34 teeth: (a) Planet gear 1, (b) Planet gear 2, (c) Planet gear 3

As in the case of LFP1 fault, Figure 11 compares the results of the extraction of the planet gear 1 TSA with two different signal shifts. The first one deals with the first maximum of the P_x and MCF functions, whilst the second one refers to a minimum in the two functions which is the farthest position of the planet gear 1 with respect to the transducer. Comparing Figures 11 (a) and (b), it is possible to see that when the planet gear is near the transducer, the engaging of the faulted tooth can be well highlighted, whereas it could be merely visible leading to poor fault identification. This result indicates that the proposed methodologies are effective for the evaluation of the planet gear position with respect to the transducer, leading to the detect of the faulty planet gear. In particular, if the real position of the planet gear with respect to the transducer is not correctly determined, the signature of the faulty planet gear cannot be suitably extracted from the noisy vibration response.

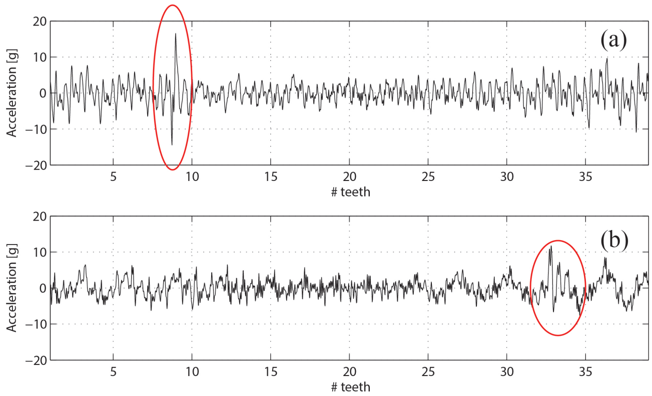

Figure 11: TSA of planet gear 1 for LFP2 fault: (a) initial phase shift of 10 teeth, (b) initial phase shift of 64 teeth

5 Concluding Remarks

In this paper two procedures for the precise evaluation of the planet gear position with respect to the transducer are presented. The first one is based on the study of the power flows inside the ring gear TSA, whilst the second method is based on a modified statistical parameter such as the Crest Factor (MCF). The effectiveness of the two methods are compared on the bases of real data.

In the first method the position of the planet gears with respect to the transducer is obtained by reconstructing the power flow at the cyclic frequency corresponding to the number of planet gears of the planetary gearbox under test. The second method relies on a modified version of the Crest Factor (MCF). In particular, the MCF is evaluated on a tooth-wide signal portion, thus obtaining a function and not a single value. The planet gears position is determined by filtering the so obtained MCF around the frequency related to the number of planet gears.

The two proposed methods give the same results, highlighting the position of the planet gears. As these positions are completely determined by the maxima of the two functions, the entire procedure could be easily automated.

From the present analysis it could be concluded that, even in the case of a particularly flat shaped transfer function of the gearbox, the evaluation of the relative position between planet gears and transducer is a pivotal information for the effectiveness of the averaging procedure. Actually, if the correct position of the planet gear with respect to the transducer is not correctly determined, the signature of the faulty planet gear cannot be extracted from the noisy vibration response.

6 Acknowledgement

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