Excitation/transfer separation in non-stationary conditions

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Abstract

This work proposes a novel approach to extend blind identification methods to signals recorded in speed-varying conditions. The excitation/transfer separation is useful for many reasons. First numerical models require knowledge of physical parameters often estimated experimentally. Moreover, for example in the case of gearboxes, the extraction of "noisy" excitation components (gear whine) enables the access to other interesting excitations. Classical blind identification methods are developed for stationary signals. In our case, the emphasis of the work is on vibration response of a mechanical system in non-stationary conditions, which makes the originality of this paper. Our technique aims to separate the excitation from the transfer in four steps by exploiting both time and angular domains. The interest is on gearbox vibration signals, modelled with a convolution between an impulse response and an excitation composed of a deterministic part depending on instantaneous "angle" frequency and a broadband random part. In the Campbell diagram in time domain (represented in speed vs. frequency in Hz) order harmonics increase proportionally to speed and structural resonances are located at constant frequencies. After angular resampling, the Campbell diagram is presented in speed vs. "angle" frequency in events/revolution. Order harmonics are then located at constant "angle" frequencies and structural resonances appear as hyperbolas. In this method the characteristics of both domains are exploited :

- In the first step the Campbell diagram is considered in angular domain and a comb filter is applied to first extract the orders.
- In a second step blind identification methods are applied in time domain to extract the impulse response, supposing that excitation is white only (harmonics being removed in the first step).
- In the third step the signal is whitened to remove the influence of the impulse response. The excitation can then be extracted in angular domain by separating deterministic and random parts (using an angle comb filter).

The validity and the limits of this approach are analysed on a gearbox signal model, with a discussion on the interest of first extracting the deterministic part of the excitation. The method is then applied on real data measured on a gearbox.

1 Introduction

Operational Modal Analysis (OMA) refers to the process for estimating modal parameters such as resonance frequencies, modal damping and mode shapes of a system in operational conditions. In this case only the measured responses are known and the inputs are unknown. A lot of methods have been developed with the assumption that the excitation is both frequentially and spatially white. Hanson [1] gave a state of the art of OMA techniques. One of them based on cepstrum is of great interest for our work. It was originally developed by Gao and Randall (1996) [2] and improved by Hanson et. al to take into account cyclostationary excitations (2007) [3]. An important point of this method is that the excitation is supposed broadband which is less restrictive than a white excitation.

This paper is focused on the estimation of transfer function of gearbox from run-up measurements. An order-based method combining Order Tracking [4] with PolyMAX modal analysis [5] is proposed by Peeters et. al (2009) [6] to identify resonances from engine run-up data.
The new method introduced in this work aims to separate the excitation from the transfer by exploiting both time and angular domains. The paper is organised as follows : the method is presented on a single-resonance model in the first part. It is then illustrated by an experimental example in the second part where an accelerometer signal placed on a gearbox in run-up condition is considered. Asumptions and model considered are finally discussed in terms of relevance and perspective.

2 Presentation of the excitation/transfer separation method

The model considered in this part is :

$$s(t) = h(t) * [d(t) + r(t)] \tag{1}$$

with

$$d(t) = \sum_k c_k(\dot{\theta}) e^{j2\pi k \frac{\theta(t)}{\Theta}}$$

where $h(t)$ is the impulse response of the system, $d(t)$ is the deterministic part of the excitation and $r(t)$ the random part. In this model the additive noise is assumed to be zero to simplify the notations but it will be taken into account in the experimental part (Eq. (11) in section 3).

The deterministic part
For gearboxes, the deterministic part corresponds to the whine noise which is principally due to the transmission error and the mesh stiffness variation. It is caracterised by a line spectrum in which most of the energy corresponds to the harmonics of the gearmesh frequency. This deterministic component is then modeled as a Fourier series depending on the input speed $\dot{\theta}$ and where Θ represents the gearmesh frequency which depends on the number of teeth of the engaged ratio in the angular domain. Gearboxes are often tested with a linear speed variation. However to present a less trivial case here one considers the following speed law :

$$\dot{\theta} = A\sin(2\pi f_s t) + Bt + C \tag{2}$$

with $A = 6$, $B = 8$, $C = 8.3$ and $f_s = 0.2$ which gives a variation from 500 to 5300 rpm in 10 s (Fig. 1 (a)). To build $d(t)$ one fixes $c_k(\dot{\theta}) = \frac{1}{k^2}\dot{\theta}(t)$ and $\Theta = \frac{2\pi}{N_1}$ with $N_1 = 43$ (modelling the number of teeth of a driving gear).

The impulse response
A mechanical system with one resonance is considered for illustrative purpose. Its impulse response is constructed from :

$$h(t) = e^{-\zeta t}\sin(2\pi f_0 t) \ \ , \ t \geqslant 0 \tag{3}$$

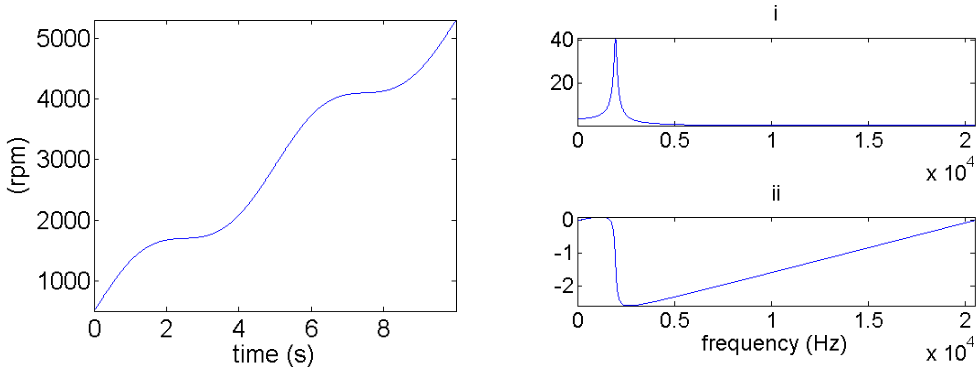
where f_0 and ζ stand for, respectively, the frequency resonance and the damping coefficient. To build a discrete minimum phase system a shift of one sample is applied on $h[n]$ (with n the discrete time variable) and the last sample is set to 0. Such a system is not physical but enables to correctly explain the method. The modulus and the phase of the transfer function are given in Fig. 1 (b).

The random part of the excitation is modeled as a white noise.

2.1 First step : extraction of the deterministic part

The first step of our method consists of the extraction of the deterministic part of the excitation which is composed of so-called ”order harmonics”.

Campbell diagrams
Figure 2 (a) presents the Campbell diagram in Hz vs. rpm of $s(t)$ in the band $[0-8000]$ Hz with a resolution of 5 Hz. In this figure the structural resonance is located at a constant frequency when speed increases



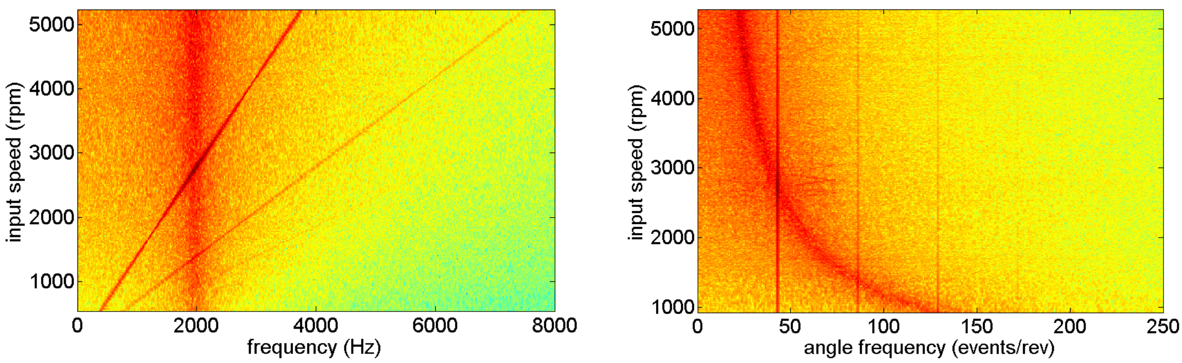
(a) (b)

Figure 1: (a) Input speed law $\dot{\theta}$. (b) Magnitude (i) and phase (ii) of the transfer function.

while harmonic orders increase proportionally to speed. An angular encoder is simulated to perform an angular resampling of $s(t)$ [7]. It gives the signal $s_\theta(\theta)$ sampled with constant angle steps. The Campbell diagram in events/rev vs. rpm of $s_\theta(\theta)$ is given in Fig. 2 (b). Order harmonics are then located at constant "angle" frequencies and the structural resonance appears as hyperbolas.

Remark : It is interesting to note that the Campbell diagrams represented in speed vs frequency differ of the time/frequency representations (idem for the angle domain). In particular order harmonics do not appear as straight lines in the time/frequency representations when the speed evolution is not linear.

Remark : The terminology of "angle frequency" is used in this paper to designate the domain of the Fourier transform of angular sampled signals. This domain should be scaled in rad^{-1}. However it is more convenient to present the results scaled in events per revolution ($events/rev$).



(a) (b)

Figure 2: (a) Campbell diagram in Hz vs. rpm of $s(t)$. (b) Campbell diagram in events/rev vs. rpm of $s_\theta(\theta)$.

Comb filter

Figure 2 (b) shows that the order harmonics can be extracted with a comb filter in the angle frequency domain, which is equivalent to a synchronous average in the angular domain.

The comb filter must have an unitary gain at the angle frequencies $f_\theta = kN_1$ (k positive integer), a null gain out of these frequencies and a null phase. One considers the following filter :

$$H_f(f_\theta) = \frac{1}{2}(1+\cos(2\pi\frac{f_\theta}{f_0}))^p \quad , \; p > 1 \tag{4}$$

where $f_0 = N_1$ and the parameter p enables to adjust the selectivity of the filter.

The Fourier transform of $s_\theta(\theta)$ is noted $X(f_\theta)$. The signal filtered in the angle frequency domain is given by :

$$F(f_\theta) = H_f(f_\theta)X(f_\theta) \tag{5}$$

The measured signal without order harmonics is then obtained with :

$$S_f(f_\theta) = X(f_\theta) - F(f_\theta)$$

An inverse Fourier transform and a return to the time domain gives access to

$$s_f(t) \simeq h(t) * r(t) \tag{6}$$

Equation (6) is not really an equality because the extraction of the orders leads also to the suppression of some parts of the response of the system when an order is equal to the resonance frequency. The Campbell diagram of $s_f(t)$ is shown in Fig. 3 and validates the extraction of the order harmonics.

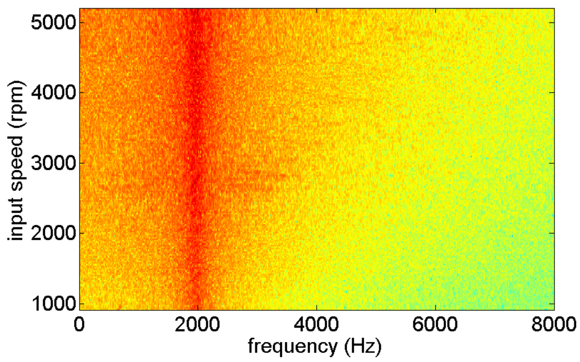


Figure 3: Campbell diagram of $s_f(t)$ in Hz vs. rpm.

The summary of accessible variables after this extraction of deterministic part is :

- $h(t) * d(t)$
- $h(t) * r(t)$

2.2 Second step : blind identification

The following step consists in identifying the impulse response of the system and then to access to its transfer function. Its knowledge is particularly useful to identify resonances of the structure, to estimate damping and to validate or update numerical models. The cepstrum is presented by Hanson [1] as an interesting tool for system identification, under the assumption that the excitation is broadband. That point is particularly useful because it is less restrictive than the white noise input assumption made by most other blind identification techniques.

Modulus identification by cepstral method

Many definitions for cepstrum are proposed in the literature. In this paper the most standard one is used :

$$\tilde{x}(\tau) = TF^{-1}[\ln|X(f)|] \tag{7}$$

Different components of a signal are differently localised in the cepstrum :

- the first sample contains the white part of the signal (as the autocorrelation of a white noise which has its whole energy at the origin),
- a broadband excitation is localised on the first samples in the low quefrencies close to the origin,

- the cepstrum of a multisine signal contains peaks on a large quefrency bandwidth and highlights its frequency,
- the impulse response of the system is localised at low quefrencies,
- the noise is distributed over the whole quefrency axis.

The first part presented in section 2.1 allows to extract the deterministic part and to obtain the signal $s_f(t) = h(t) * r(t)$. Its cepstrum $\tilde{s}_f(t)$ is presented in Fig. 4.

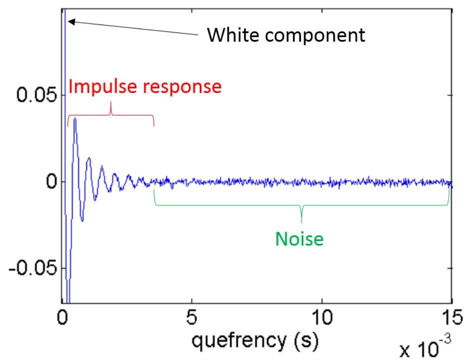


Figure 4: Cepstrum of $s_f(t)$.

An estimation of the cepstrum of $h(t)$ is obtained by setting to 0 the first sample of $\tilde{s}_f(t)$ (corresponding to the white random excitation) and by smoothing the quefrencies containing only noise (here estimated up to 500 samples).

Using definition (7), the modulus of the transfer function is estimated by :

$$|H(f)| = e^{TF[\tilde{h}(t)]} \tag{8}$$

The estimated modulus is compared to the theoretical modulus of the transfer function of the system in Fig. 5 (a), both normalised with unit energy and limited in the band $[0-8000]\,Hz$. The modulus is well estimated: the resonance frequency is well recovered in terms of frequency and magnitude. The estimated modulus is noisier because the low quefrencies contain also some noise (they do not contain only the impulse response).

Phase estimation

From the knowledge of the modulus, the phase can be estimated by supposing a minimum phase system, that is to say that the system and its inverse are causal and stable. By construction the system presented in this section is minimum-phase. Moreover in the rest one will suppose that the mechanical systems of our interest can be considered as minimum-phase.

The minimum phase assumption implies that the modulus and the phase of the transfer function are related by the Hilbert transform [8] :

$$\Phi(f) = -TH[\ln |H(f)|] \tag{9}$$

The phase thus estimated is compared to the theoretical phase of the transfer function of the system in Fig. 5 (b) in the band $[0-8000]\,Hz$. It shows that the phase is well recovered by this method.

Impulse response

By inverse Fourier transform one obtains the impulse response estimated with our method (Fig. 5 (c)). The impulse response is then well recovered.

This step of blind identification enables to recover a good magnitude and phase of the transfer function and then to identify some resonances and damping of the structure and can be used to "whiten" the signal.

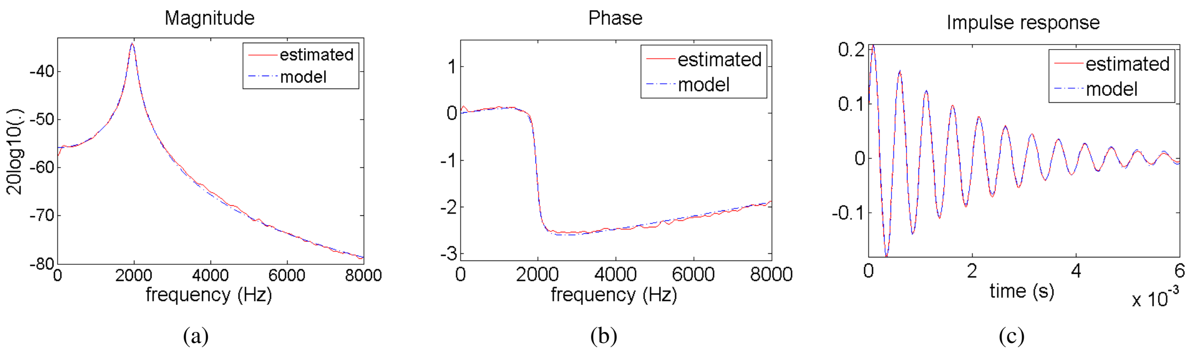


Figure 5: Comparison of the magnitude (a), the phase (b) and the impulse response (c) estimated and theoretical. Magnitudes and impulse responses are normalised to unit energy.

2.3 Third step : whitening

The estimation of $H(f)$ enables to whiten the signal : it consists of removing the influence of the transfer on the signal. The inverse filter $H^{-1} = \dfrac{1}{H}$ is classically used. By construction this filter is here stable and causal (minimum-phase assumption). Such a filter has huge values at low frequencies where $|H(f)|$ has some values close to 0, which will amplify low frequency components of the whitened signal. An alternative would be to use an inverse adjusted filter [9] :

$$H^{inv} = \frac{H^*}{|H|^2 + f_b} \tag{10}$$

where f_b is a noise factor which enables to compensate amplification inequalities on the whole frequency band. However this filter is not causal.

Figure 6 presents the Campbell diagram of the signal $s_b(t) = s(t) * h^{-1}(t)$ whitened with the classical inverse filter. A good equalisation is observed on the whole frequency band, the system resonancy has been removed. In this case the amplification of low frequencies by the classical inverse filter is not problematical and the use of the adjusted filter does not bring improvement. Moreover in non-stationary conditions the amplitude of vibrations varies depending on speed. In this model a proportional variation with the speed is considered, which is visible in Fig. 6. The signal could also be whitened in this sense but we are not interested here by such a method.

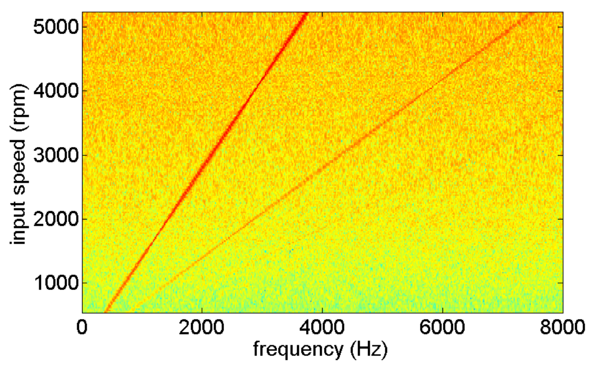


Figure 6: Campbell diagram of the withened signal $s_b(t) = s(t) * h^{-1}(t)$.

This whitening step gives access to the excitation $e(t)$ isolated from the transfer.
By applying a comb filter on $e(t)$ the deterministic and random parts can be separated and one access to an estimation of the following variables :

- $d(t)$
- $r(t)$

2.4 What happens if one does not extract the orders before the blind identification ?

The first step of this method allows to extract the deterministic part of the excitation before applying blind identification techniques. For the non-stationary cases considered, one might think that the orders excite the structure as a white excitation because they scan all the frequencies. To test this assumption the method is applied without extracting the orders (that is to say without applying the first step of the method). The magnitude and the phase thus obtained are compared to the theoretical ones in Fig. 7. One observes that the estimation of the magnitude and the phase is not correct, some faked resonances are added. It shows that the orders cannot be considered as a white excitation. This point was to be expected because orders do not excite simultaneously all the frequencies.

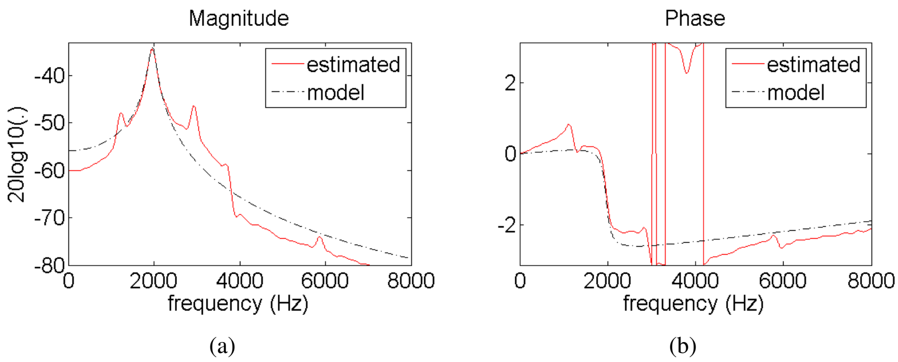


Figure 7: Comparaison of magnitude (a) and phase (b) estimated without extraction of the orders and theoretical ones. Magnitudes are normalised to unit energy.

3 Application on a measured signal

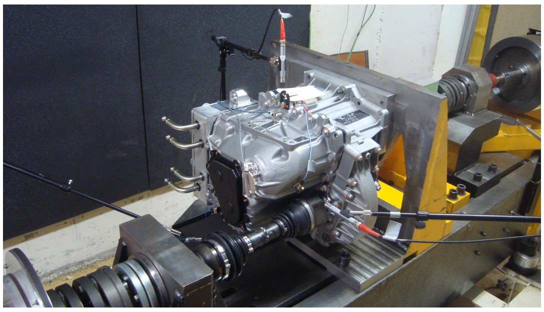
The method to separate the excitation and the transfer components of a signal in non stationary conditions is applied to a vibratory signal measured on a gearbox.

3.1 Presentation of the test bench

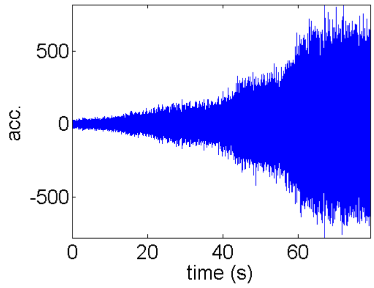
The test bench used in this study is composed of an automotive gearbox driven by an electrical motor which is speed monitored. An acyclic excitation can be applied by means of a Cardan joint. A loading is applied to the gearbox by an electrical generator which is torque monitored. The gearbox is equiped with different transducers (accelerometers and microphones) as shown in Fig. 8 (a). The third gear is engaged (gear ratio $43/56$) and a speed ramp is applied. An optical encoder with a resolution of 60 pulses per revolution is positioned on the bench between the electrical motor and the input of the gearbox.

One accelerometer signal is considered (Fig. 8 (b)) .The conditions of the measurement are :

- speed ramp from 750 rpm to 3400 rpm in 79 s,
- angle of cardan joint of 6°,
- sample frequency : 25600 Hz.



(a)

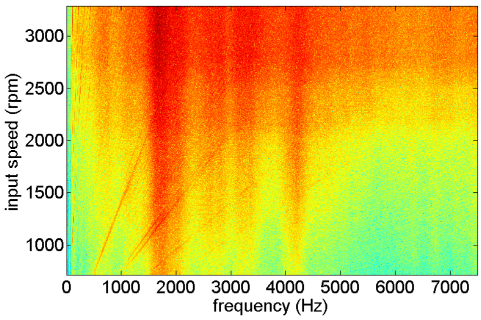


(b)

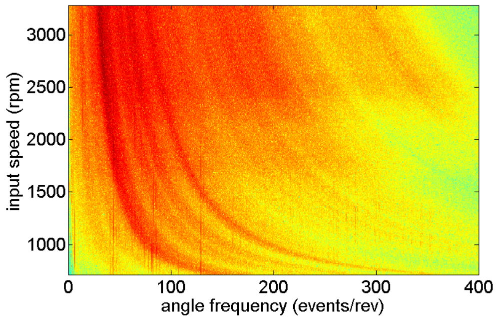
Figure 8: (a) Gearbox on the test bench. (b) Gearbox vibration signal.

3.2 Results and discussion

Figure 9 (a) shows the Campbell diagram of the accelerometer signal. The order harmonics linearly increase with speed and structural resonances are located at constant frequencies. After angular resampling the order harmonics are then located at constant angle frequencies and structural resonances appear as hyperbolas (Fig. 9 (b)).



(a)



(b)

Figure 9: (a) Campbell diagram in Hz vs. rpm. (b) Campbell diagram in events/rev vs. rpm.

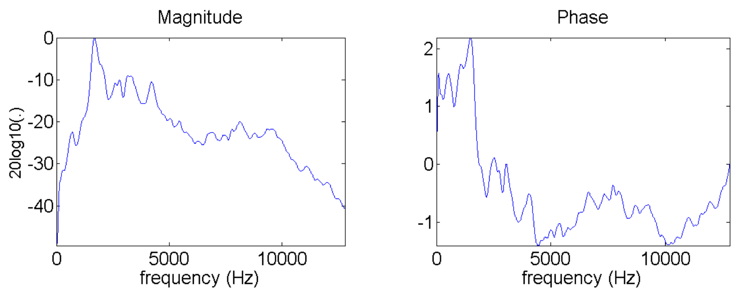
The method presented in the previous section is applied on the measured signal $s(t)$ of supposed form

$$s(t) = h(t) * [d(t) + r(t)] + n(t) \tag{11}$$

with $n(t)$ the additive noise. After the extraction of the deterministic component $d(t)$ with a comb filter a SISO signal is considered (with $r(t)$ a white excitation). Moreover one supposes that the load is constant and then that $h(t)$ is the same for the entire measurement (whatever the speed). These hypothesis will be discussed in the next paragraphs.

Figure 10 presents the estimated transfer function $H(f)$. To observe the influence of the speed, the magnitude is estimated on 3 different portions of the signal (Fig. 11). The main resonances are globally localised at the same frequencies for the 3 speed cases but their amplitudes are different. At the low frequencies the magnitude level is higher for the low speeds than for the high speeds and in the high frequencies the amplitude increases with the speed. The difference between the levels reaches about 15 dB around 10 kHz. Such a difference could be rather attributed to some variations in the excitation than only on the transfer function. The excitation seems to be more high frequency when the speed increases.

The estimated transfer function is now used to whiten the measured signal. A comparison between the Campbell diagrams of the measured and whitened signal is presented in Fig. 12. The low frequency components are amplified by the inverse filter used to whiten the signal as mentioned in section 2.3. A huge part of the resonances are removed by the whitening but there are still residues remaining. These residues could be



(a) (b)

Figure 10: (a) Magnitude and (b) phase of the estimated transfer function.

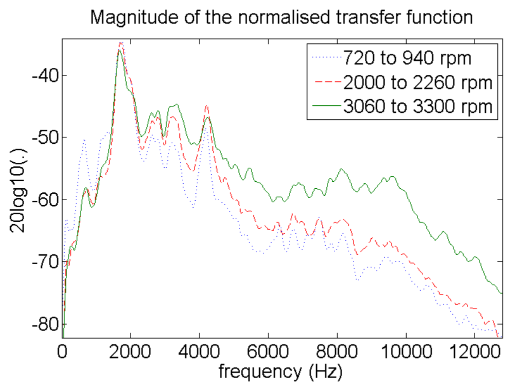
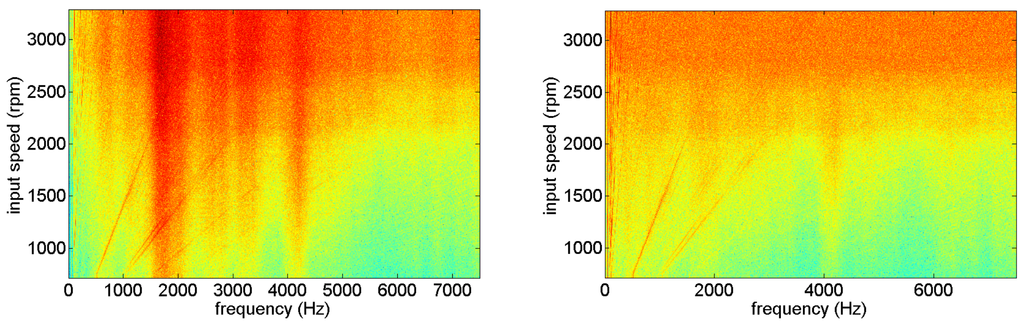


Figure 11: Comparaison of the magnitude of the transfer function estimated on 3 different sections of the signal (blue ... : 720 to 940 rpm, red – : 2000 to 2260 rpm, green : 3060 to 3300 rpm).

attributed to the simplicity of the considered model where a white excitation is convolved to an unique impulse response. Figure 11 shows that the excitation must have different amplitude levels according to the speed and the frequencies. Moreover according to Fig. 12 (b) one should rather consider a Multiple Input Single Output signal $s(t) = \sum_i h_i(t) * r_i(t)$. A perspective of this work is to extend the method to such a signal model.



(a) (b)

Figure 12: (a) Campbell diagram in Hz vs. rpm of the measured signal. (b) Campbell diagram in Hz vs. rpm after whitening with the estimated $H(f)$.

4 Conclusion

The proposed method performs a separation between transfer and excitation based on the exploitation of both time and angular domains in non-stationary conditions. An excitation composed of a deterministic part (the orders) and a random part is considered. After extraction of the orders a SISO system is assumed and a blind identification based on cepstrum is applied. It is shown that the extraction of the orders is essential before applying blind identification techniques even in severe run-up conditions. With the assumption that the system is minimum-phase the modulus and the phase of the transfer function is recovered. It is then used to whiten the signal by removing the influence of the transfer. The application on a gearbox shows that the consideration of a SISO system might not be sufficiently representative.

A future direction of research is to consider a system with multiple inputs and multiple transfers.

Acknowledgements

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