

# Diagnosis of gear faults by cyclostationarity

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## Abstract

Gearbox maintenance is not an easy task and conventional diagnostic techniques do not always provide suitable indication about failures. In this paper, tooth crack failures have been experimentally investigated. The mechanical system is composed of a two-stage gearbox with spur gears. Two depths of gear cracks are treated. Cyclostationarity analysis of order 2 has been conducted to some experimental signals delivered by a test bench under two loads by using an electromagnetic brake. The information has been extracted by using the squared envelope of the signal. The results show that the cyclostationarity approach is advisable for extracting the amplitude variation at the characteristic fault frequencies related to the defect. Experimental applications for diagnosis of tooth crack demonstrate that this technique is powerful and effective in feature extracting and fault detection for gearboxes. The crack signature was shown very sensitive to the crack severity whatever the applied load.

## 1 Introduction

The mechanisms with gears play an important role in various industrial applications. Gearbox transmission systems are one of the fundamental and most important parts of machinery and are employed in industrial sectors worldwide. Gearbox transmissions may undergo excessive stresses that led to wear and malfunctions, the appearance of defects and consequently may stop production. Frequently working in severe conditions, gears are subjected to several types of defects, especially on their teeth [1], and their maintenance is conditioned to an adequate monitoring. The early detection and diagnosis of gear faults is therefore crucial to prevent the system from malfunctions that could cause damage or entire system halt [2]. Up to now, fault diagnosis of industrial gearboxes has received intensive study for several decades, and vibration analysis of these machines can detect certain types of defects (unbalance, misalignment, blade pass, friction, noise, fluctuation, turbulence, etc.) [3]. Determination of each of these types of faults constitutes in itself a powerful monitoring technique. Some monitoring methods applied to gears are simple to use like the scalar descriptors (RMS, Peak, Kurtosis, etc.) [4] and spectral analysis (Fast Fourier Transform, envelope analysis, cepstrale analysis, etc.) [5]. However these methods are not able to distinguish between the nonlinear and the nonstationary behaviours due to the fluctuation of the velocity and load or the random phenomena (variation of wear on the surface of teeth or the opening and closing of cracks). The diagnosis consequently requires more sophisticated signal processing techniques. It is important to determine the dynamic loads on all their components (axis, teeth, bearings...), the level and variation of local stresses in these components or identify transient phenomena.

In this paper, the theory of cyclostationary is applied to our model as a tool for the diagnosis of gear defects since there are a combination of periodic and random processes due to the machine's rotation cycle and interaction with random phenomenas [6]. This approach is chosen because it is very well suited for distinguishing the faults of rotary machines for several reasons [7]: first, the occurrence of a fault in a rotating component will typically produce a repetitive variation of vibration energy, secondly because the cyclostationary framework makes it possible to localise precisely a fault within the machine kinematics and

thus simplifying its detection [8-11]. The paper is organized as follows: the cyclostationary statistical method is introduced in the section 2. In section 3, the experimental description of the test bench and a real cracked tooth will be investigated. Finally, we will compare and discuss the different results obtained from the real signals and the advantage of cyclostationarity in the diagnosis of gear cracks.

## 2 Cyclostationarity analysis

During the industrial development at the last decades, several signal processing techniques have been adopted to diagnose the faults of machinery. Among all these techniques, cyclostationary analysis is well suited for detecting defects that comes from cyclical or repetitive phenomena in rotating machinery. Generally, the mechanical systems seldom produce cyclostationary signals of order 1 or 2 but rather a combination of the two orders. So, it is very advantageous to analyze these two types of cyclostationarity separately because they are likely to be generated by phenomena of different physical origins. The order 1 is rather resulting from phenomenon of determinist nature (unbalances, eccentricities, meshing 'gear faults') whereas the cyclostationarity of order 2, CS2, is resulting from phenomena of random nature (impacts, frictions,...) [10]. In general, cyclostationarity uses an extraction operator " $P\{\circ\}$ ". This operator extracts all the periodic components in a time function. This operator is defined as:

$$P\{\circ\} = \sum_{\alpha \in A} P_{\alpha}\{\circ\} \quad (1)$$

where

$$P_{\alpha}\{\circ\} = \left( \lim_{T \rightarrow \infty} \frac{1}{T} \int_T \circ \cdot e^{-j2\pi\alpha t} dt \right) \cdot e^{+j2\pi\alpha t} \quad (2)$$

$\alpha$  is commonly known as the cyclic frequency of the signal and its inverse as the cycle and  $P_{\alpha}\{\circ\}$  is the operator that extracts the periodic component of the signal at frequency  $\alpha$ .

When applied to signal  $x(t)$ , it extracts the perfectly predictable part of the time function. When applied to the energy flow  $P\{|x(t)|^2\}$ , it reveals the repetitive parts of energy that characterize the presence of a periodic mechanism. If the energy flow is suspected to vary periodically, the  $P\{|x(t)|^2\}$  will have a Fourier series decomposition with non zero coefficients, say  $P_x^{\alpha}$ , which denotes the cyclic power of the signal at the cyclic frequency  $\alpha$ .

$$P\{|x(t)|^2\} = \sum_{\alpha \in A} P_x^{\alpha} \cdot e^{j2\pi\alpha t} \quad (3)$$

Generally, these quantities are decomposed in frequency domain using the spectral correlation given by equation (21).

$$CS_x^{\alpha}(f) = \lim_{\Delta f \rightarrow 0} \frac{1}{\Delta f} P_{\alpha} \left\{ x_{\Delta f} \left( t; f + \frac{\alpha}{2} \right) \cdot \overline{x_{\Delta f} \left( t; f - \frac{\alpha}{2} \right)} \right\} \quad (4)$$

where  $x_{\Delta f}(t; f)$  is the filtered version of the signal  $x(t)$  through a frequency band of width  $\Delta f$  centred on frequency  $f$   $\left( \left[ f + \frac{\Delta f}{2}, f - \frac{\Delta f}{2} \right] \right)$ .

The spectral correlation is a function of two frequency variable  $f$  and the cyclic frequency  $\alpha$ . A non zero value of the spectral correlation translates the existence of a package of waves carried by the frequency  $f$  whose energy fluctuates periodically at the frequency  $\alpha$ . It is important to indicate that the evolution of the spectral correlation for  $\alpha = 0$  is identified with the power spectral density (PSD) of the presumed stationary signal. Its integration along the axis of the carrier frequencies is identified with the cyclic coefficients of the instantaneous power and finally its summation in series of Fourier makes it possible to define an energy distribution along the two axes time and frequency [8] (called Wigner-Ville spectrum), see equation (5).

$$WV_x(t, f) = \sum_{\alpha \in A} CS_x^{\alpha}(f) \cdot e^{j2\pi\alpha t} \quad (5)$$

The periodic part of the signal is simply calculated by performing a cycle synchronous averaging  $P\{x(t)\}$ . The residual part is extracted by subtracting the periodic part from the original signal ( $x^r(t) = x(t) - P\{x(t)\}$ ). A non zero value of  $P\{|x^r(t)|^2\}$  or in general a non zero value of the spectral correlation of the residual signal translates the existence of cyclostationarity of second order. Unfortunately, speed fluctuations cause smearing in spectrums. Also these signals are not purely periodic. Consequently the time synchronous averaging fails to separate the periodic part from the residual part of the signal. To do this, R.B. Randall *et al* [12] have proved that the envelope analysis (and even squared envelope) gives the same results as the integration of the cyclic spectral density function over all frequencies, thus establishing the squared envelope analysis as a valuable tool for the analysis of (quasi-) cyclostationary signals, see Fig. 7.

$$\hat{M}_{xx}(\alpha) = \lim_{W \rightarrow \infty} \frac{1}{W} \int_{\frac{W}{2}}^W |x(t)|^2 e^{-j2\pi\alpha t} dt = \lim_{W \rightarrow \infty} \frac{1}{W} \int_{\frac{W}{2}}^W C_{xx}(t, 0) e^{-j2\pi\alpha t} dt \quad (6)$$

where the integral has been divided by duration  $W$ .

If the analysed vibration signal is dominated by its stochastic part, then:

$$C_{xx}(t, 0) = E\{|x(t)|^2\} \quad (7)$$

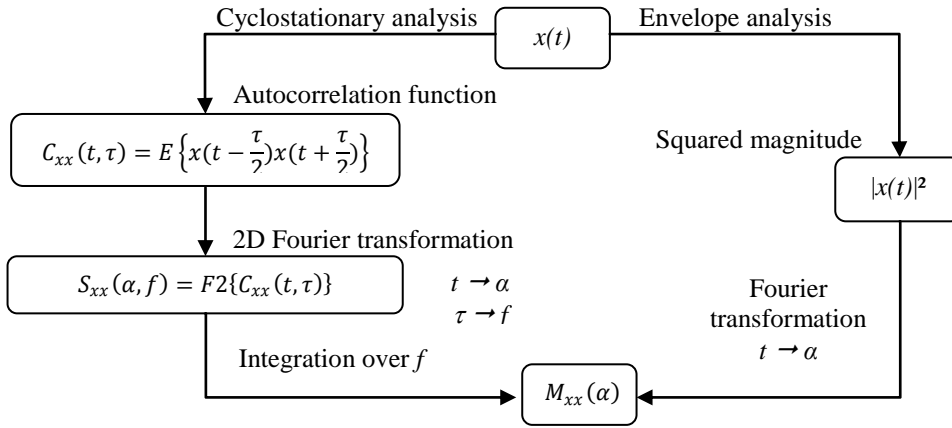


Figure 1: Equivalence between the integrated cyclic spectral density and the spectrum of the squared amplitude signal

This approach will be used in the next section in order to process the cyclostationarity of 2nd order of the signals acquired from defected gears.

### 3 Experimental analysis

#### 3.1 Description of the test bench and experimental protocol

The test bench consists of a two stage toothed gear hooded by a structure made of steel plate (see Fig. 2). The bearing housing and the lid are made of Aluminium to allow vibration measurements. The four gear wheels have straight teeth with a modulus of 1 for the input gear and 3 for the output gear. The number of teeth is  $Z_1= 88$ ,  $Z_2= 90$ ,  $Z_3= 29$  and  $Z_4= 30$ , respectively. Bearings with one line of balls (SKF 6002 type) are assembled on the six housings. The gears are immersed in a lubricant with a well-defined viscosity (ISO VG 220, VI 95) and its temperature is controlled by a heating element and a PID controller. The gears are driven by an asynchronous motor and in the output are linked to an electromagnetic brake as a variable load. In each gear train, the number of teeth is very close which makes the reduction ratio very close to 1. The sampling frequency was set at 48 000 Hz and the max data acquisition time was set at 10s in order to increase the frequency resolution (0.1 Hz) and separate the frequencies of three shafts. The accelerometer was placed on the bearing housing of the output, in the direction of the line of action of the output gear.

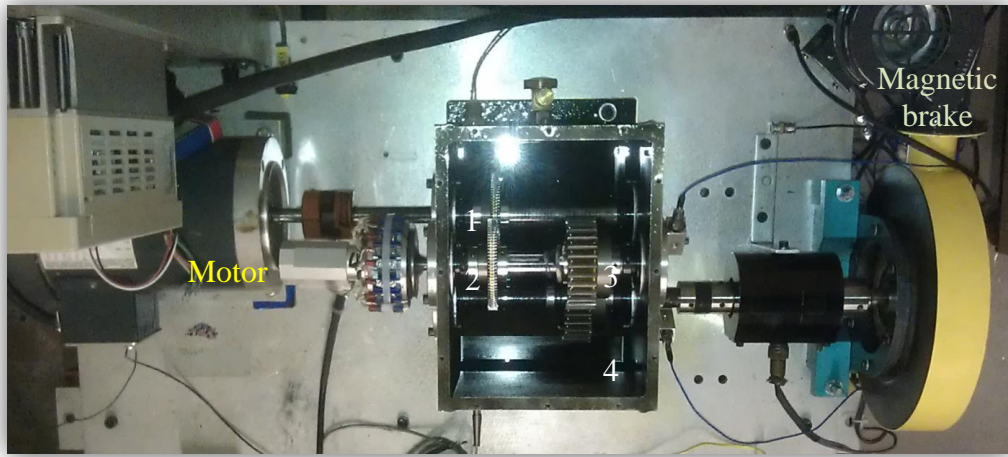


Figure 2: Test bench (two stage speed reduction gear ).

Two types of degradation were imposed on a wheel: a crack of 33% of the tooth width and another of 100%. Both defects have the same depth (1 mm), see Fig. 3. The tests were performed at the rotating speed of 2040 rpm and at a temperature of 30°. The results are compared for two loads (3 *N.m* and 5 *N.m*).

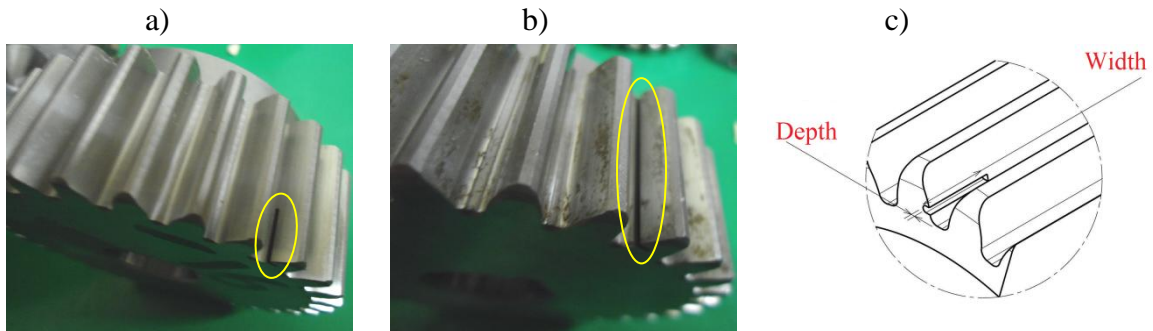


Figure 3: Crack dimensions: a) 33% of the tooth width, b) 100% of the tooth width, c) Crack model.

### 3.2 Experimental results

Fig. 4 shows the time signal of the two cracked wheels. It is very difficult to carry out a robust diagnostic from these figures.

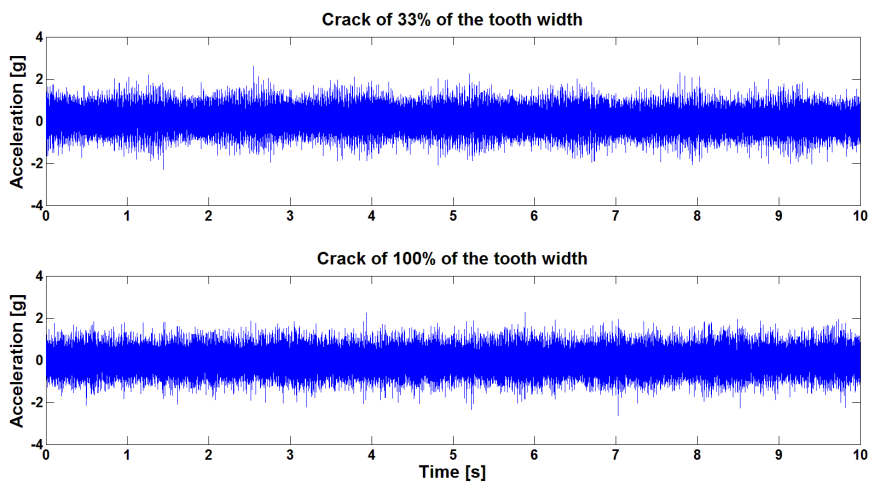


Figure 1: time signals of cracked teeth.

Fig 5 shows the corresponding spectrum under two different loads (3 & 5 N.m).

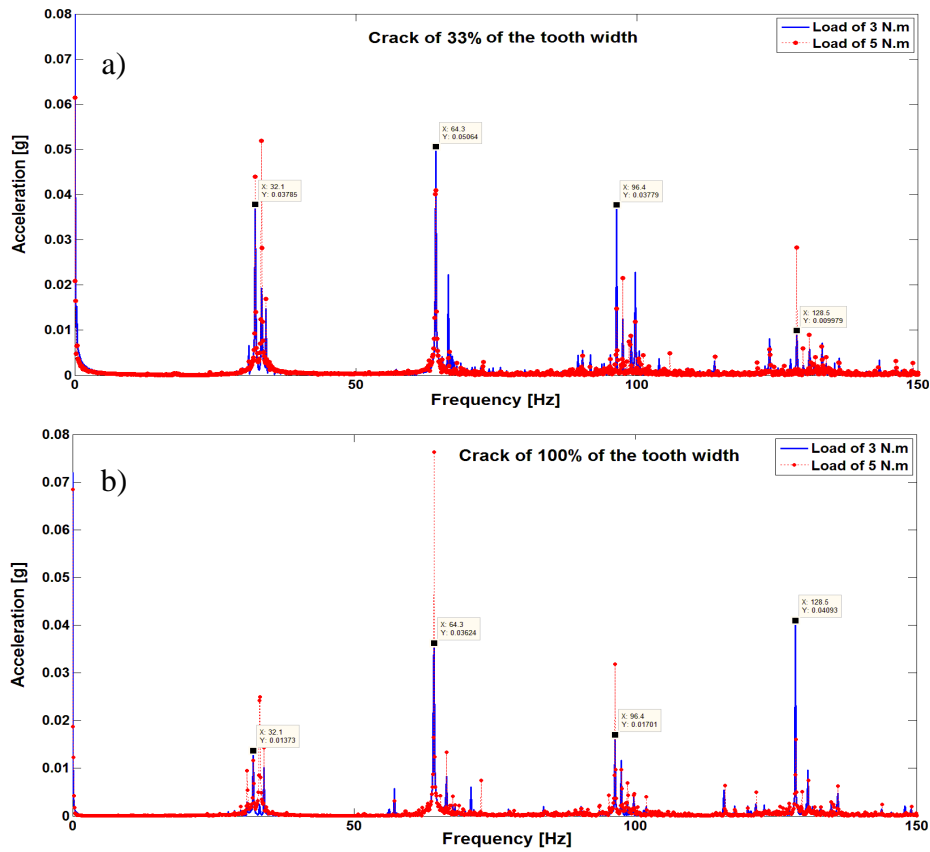


Figure 2: FFT of signal: a) tooth with crack of 33% b) tooth with crack of 100%.

Fig 5 shows the difficulty for diagnosing a crack, the amplitudes in both crack severity being similar. When the crack is important (100%: Fig 5b), it can be noticed that all the amplitudes at the harmonics the gear mesh frequency are greater than the amplitude at its fundamental whatever the load. It is a descriptor of defect [3]. It is also true for a smaller crack (33%) under a small load (3N), but not when increasing the load (Fig 5a). In order to apply cyclostationarity, we have used the angular resampling technique to simplify the calculation and the extraction of the synchronous average (Fig. 6. a). Secondly, the subtraction of the synchronous average from the raw signal  $x(t)$  gives the residue or the variance of the signal (Fig. 6. b) that is carrying the moments of 2nd order. The FFT of the angular variance (Fig.6. c) reveals that the harmonics of the gear mesh frequency and its fundamental frequency with modulations.

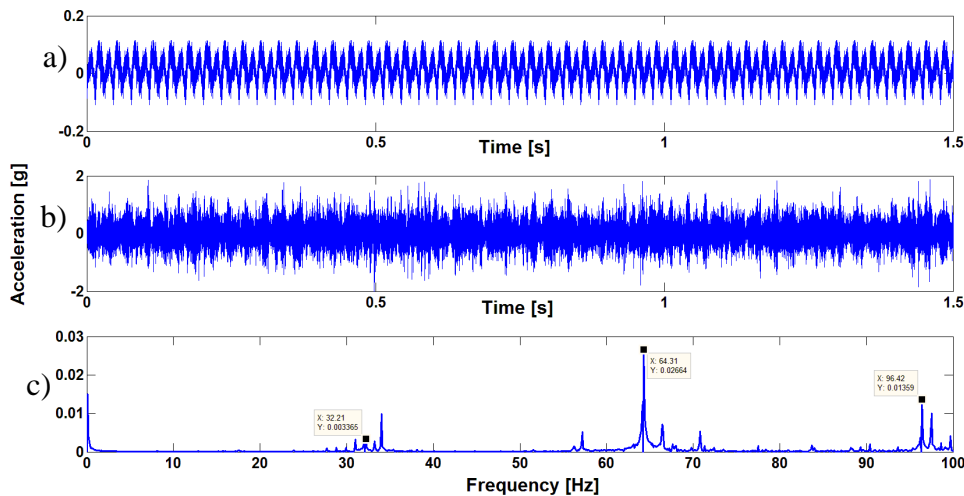


Figure 3: Calculation of cyclostationarity moments: a) synchronous average b) variance c) FFT of variance .

After applying the theory of the envelope analysis to real signals to find the equivalent of the cyclic spectral density CSD, we notice in Fig. 7.a. the presence of the shaft frequency that carries the defect gear (32,1 Hz) and its harmonics, but with a very low amplitude for both load (3 & 5 N.m) in the case of a crack of 33%. However, in the case of crack of 100%, the amplitudes at the cyclic frequency of the shaft which carries the fault  $\alpha = 32,1$  Hz increases whatever the load. When increasing the load, the amplitude at the second harmonic increases strongly (Fig. 7. b). These figures show the sensitivity of cyclostationnarity of second order in the diagnosis of defects.

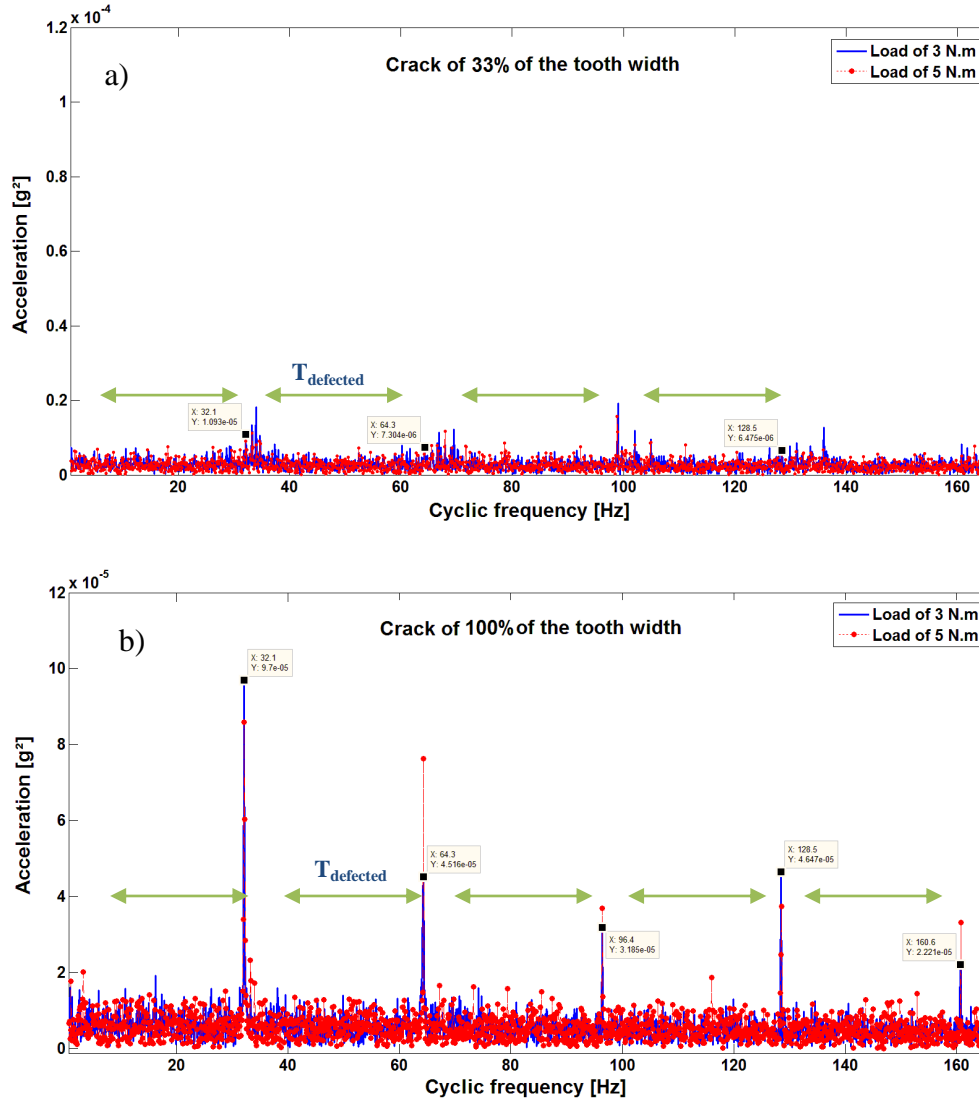


Figure 4: CS2 extracted by the squared envelope approach: a) tooth with crack of 33% b) tooth with crack of 100%.

## 4 Conclusion

Cracked gear signals contain a deterministic part due to the rotation of the shaft and a stochastic part due to the random fluctuation caused by the load, machining error, random fluctuations of stiffness, etc. Consequently, cyclostationnarity analysis seems a good candidate to analyse its dynamic behaviour. This study was aimed to evaluate the feasibility to use cyclostationnarity of CS2 for detecting gear cracks in order to extract the parameters and hidden information in the signal. The classical way to calculate the cyclostationnarity of second order is the subtraction of the synchronous average from the raw signal but if the treated signal contains components with frequencies very close with phase fluctuations, the separation of the statistical moments may become complicated. To overcome this problem, the use of the squared envelope of the signal of the cracked wheel proved to have a high efficiency for the calculation of the CS2.

Cyclostationarity has been applied to experimental signals coming from two damaged tooth crack. The results show that these information may be used for condition monitoring. In our study, it has been shown that cyclostationarity of CS2 can easily distinguish the severity of crack while it is difficult by using the usual Fast Fourier Transform applied to the original signal.

In the future, we will develop a numerical model of damaged gears in order to numerically investigate the sensitivity of cyclostationarity to various conditions of operation and defects and prove its effectiveness to monitor machinery vibration.

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