Application of Speed Transform to the diagnosis of a roller bearing in variable speed

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Abstract

In constant speed operation, vibration signals of rotating machinery and their statistics usually exhibit periodicities. These periodicities can be brought out through Fourier analysis, by applying either spectral analysis or cyclostationary analysis. In variable speed operation, all the components whose frequencies are tied to the speed of the machine follow the speed variations. Looking for constant frequency oscillatory functions within the signal is not as relevant as in constant speed operation. Since many diagnosis tools have been developed based on Fourier transform, it is worth looking for equivalent tools in variable speed. The usual techniques consist either in segmenting the signal into slices short enough for the speed to be constant, or in re-sampling the signal so that the samples correspond to fixed angular positions rather than being temporally equi-spaced. The first method’s drawback is that the shorter the slices, the poorer the resolution of frequency analysis, while the second method’s drawback is that by resampling the signal, one alters the resonance phenomena, that are not tied to the speed of the shaft.

Here, we propose to adapt Fourier transform to the vibration signal, rather than altering the signal for it to be adapted to Fourier analysis. The new tool that we propose, called Speed Transform, consists in decomposing the signal over a basis of elementary oscillatory functions whose frequencies follow the speed variations. It has been shown in [1] to have equivalent properties to Fourier transform, provided that the speed is linearly varying. It can be applied to the signal without any prior resampling or segmentation, which allows extending to variable speed operation most of the classical tools based on Fourier analysis for the constant speed operation. In this paper, Speed Transform is applied to the vibration signal of a roller bearing. We show how it allows computing under variable speed operation classical parameters such as envelope spectrum and spectral correlation, which then become envelope Speed Transform and Speed correlation.

1. Introduction

During many years, vibration analysis techniques have been applied in constant speed operation, in order to ensure that the vibration signals were stationary or cyclostationary. But it is not always possible to get some records performed in such peculiar conditions. Indeed, the machinery rotation speed cannot be set according to the surveillance needs. Furthermore, some damages can be revealed in non stationary operation, for example by exciting some resonance during a speed-up. For these reasons some techniques have been developed lately for the analysis of vibration signals in non stationary operation. In stationary operation the Fourier transform is widely used to put in evidence some repetitive phenomena tied to the presence of a damage on a rotating part. In non stationary operation, tracking strictly periodic components is not relevant any more. Among the techniques adapted to variable rotation speed, some are based on filtering, but others still rely on Fourier transform, though requiring some alteration either of the signal or of the analysis tool. One can split the signal into slices and suppose the rotation speed is constant over the duration of a slice, or resample the signal for it to be expressed versus rotation angle rather than time. The drawback of the first technique is a worsened frequency resolution, while resonance phenomena are altered by the latter. Some new techniques consisting in a modified transform have been proposed. The "Velocity Synchronous Discrete Fourier Transform" [2] performs angular resampling and Fourier transforming at the same time, while the "Time Variant Discrete Fourier Transform" [3] is a modified (non-orthogonal) Fourier transform whose kernel follows the speed variations. The tool applied here consists in calculating a Fourier transform whose kernel follows speed variations over a duration sufficient to ensure orthonormality, provided that the speed variations are linear. In section 2 we first study what happens to roller bearing vibrations in variable rotation speed through calculations performed over a simplified model. We deduce from these calculations which components of the autocorrelation function follow the speed variations and propose to extend classical analysis techniques such as envelope spectrum and spectral correlation through the use of Speed Transform instead of Fourier Transform. Two new analysis techniques are derived, called Envelope Speed Transform and Speed Correlation, and applied to simulated signals fitting to the simplified model. In section 3 these new tools are applied first to a simulated roller bearing vibration signal and then to a real-life one.

2. Simplified model of the bearing vibration signal in variable speed operation

2.1 Signal model in constant speed operation

In constant speed operation, the vibration signal of a damaged roller bearing is mainly produced by shocks that excite the mechanical structure. These shocks occur at periodic intervals, the periodicity being altered by some jitter due to the movements of the rollers within the cage. Such a model can be described as follows [4].

$$s(t) = m(t)\big(\delta_{Td}(t) * h(t)\big) \tag{1}$$

- ✓ $m(t)$ is an amplitude modulation, periodic at the rotation speed frequency,
- ✓ T_d is the period of the shocks,
- ✓ $\delta_{Td}(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT_d)$ is a pulse train standing for the exciting shocks. Some random jitter can affect the pulse times nT_d,
- ✓ $h(t)$ is the response of the mechanical structure.

This can be viewed as a sum of complex exponentials with complex random amplitudes. The frequencies of these periodic waves are all the possible combinations of the damage shocks frequency ($f_d = 1/T_d$) harmonics and the rotation frequency (f_r) harmonics $f_{k_1,k_2} = k_1\, f_d + k_2\, f_r$ with $\{k_1, k_2\} \in Z^2$ due to the modulation $m(t)$. The vibration signal can thus be written as follows.

$$s(t) = \sum_{k_1=-\infty}^{+\infty} \sum_{k_2=-\infty}^{+\infty} a_{k_1,k_2} e^{2\pi j\, f_{k_1,k_2}\, t} \tag{2}$$

The amplitudes a_{k_1,k_2} are complex and random. They take into account:

- ✓ the resonance of the structure,
- ✓ the phases of the oscillatory functions,
- ✓ the jitter on the shocks period.

These amplitudes are usually correlated, due to the fact that all these oscillatory components are produced by the same physical phenomenon. As a consequence, both envelope spectrum and spectral correlation exhibit spectral lines when applied to such a signal [5]. They thus are classically used as analysis tools in order to detect the characteristic shocks produced by any damage and diagnose the roller bearing.

2.2 Extension of a simplified model to variable speed

In order to study from a theoretical point of view what happens in time-varying operation, the autocorrelation of the signal should be calculated. Indeed, the spectral correlation is obtained from the autocorrelation function by applying Fourier transform over both time and time-lag, while the envelope spectrum is obtained by applying Fourier transform over time for zero time-lag. A simplified model given in Eq (3) will be studied here in order to avoid heavy calculations.

$$s(t) = a_1 e^{2\pi j \int_0^t f_1(u)du} + a_2 e^{2\pi j \int_0^t f_2(u)du} \tag{3}$$

The respective complex random amplitudes a_1 and a_2 of the two components can be correlated to each other or not. Their respective frequencies are time-varying, For instance, they can follow the rotation frequency variations.

The autocorrelation function is defined by $R_s(t,\tau) = E[s(t)s^*(t-\tau)]$ where E[...] stands for ensemble averaging and * for complex conjugate. The calculation of the autocorrelation of the signal defined by Eq. (3) is given in appendix. In what follows, the frequencies of the two components are supposed to be linearly varying and equal to $f_1(t) = \alpha_1 t + \beta_1$ and $f_2(t) = \alpha_2 t + \beta_2$.

In the peculiar case of zero time-lag ($\tau = 0$) it is equal to:

$$R_s(t,0) = E\{|s(t)|^2\} = E\{|a_1|^2\} + E\{|a_2|^2\} + 2\, Re\{a_1 a_2^*\} cos\left[2\pi\left(\frac{(\alpha_2-\alpha_1)}{2}t^2 + (\beta_2-\beta_1)t\right)\right] \tag{4}$$

The autocorrelation function depends only on time and follows the variations of $\alpha_0(t) = f_2(t) - f_1(t)$. In constant speed operation, it would be periodic at a fixed frequency $\alpha_0 = f_2 - f_1$ and the signal would be cyclostationary at frequency α_0. The envelope spectrum would exhibit a spectral line at cyclic frequency α_0. We propose to extend this technique to the variable speed case by applying to the signal an Envelope Speed Transform (EST). Provided that both $f_1(t)$ and $f_2(t)$ are proportional to the rotation frequency variations $f_r(t)$, the EST should exhibit an order line at the order K such that $\alpha_0(t) = K f_r(t)$.

In the most general case the autocorrelation can be decomposed into two auto-terms $R_1(t,\tau)$ and $R_2(t,\tau)$ and two cross-terms $R_{2,1}(t,\tau)$ and $R_{1,2}(t,\tau)$, whose expressions are given below.

$$\begin{cases} R_1(t,\tau) = E\{|a_1|^2\} e^{2\pi j\left(\alpha_1 t\tau + \beta_1\tau - \frac{\alpha_1}{2}\tau^2\right)} \\ R_2(t,\tau) = E\{|a_2|^2\} e^{2\pi j\left(\alpha_2 t\tau + \beta_2\tau - \frac{\alpha_2}{2}\tau^2\right)} \end{cases} \tag{5}$$

$$\begin{cases} R_{2,1}(t,\tau) = E\{a_2 a_1^*\}\, e^{2\pi j\left(\frac{(\alpha_2-\alpha_1)}{2}t^2 + (\beta_2-\beta_1)t\right)}\, e^{2\pi j\left(-\frac{\alpha_1}{2}\tau^2 + \beta_1\tau\right)}\, e^{2\pi j \alpha_1 t\tau} \\ R_{1,2}(t,\tau) = E\{a_1 a_2^*\}\, e^{2\pi j\left(\frac{(\alpha_1-\alpha_2)}{2}t^2 + (\beta_1-\beta_2)t\right)}\, e^{2\pi j\left(-\frac{\alpha_2}{2}\tau^2 + \beta_2\tau\right)}\, e^{2\pi j \alpha_2 t\tau} \end{cases} \tag{6}$$

The two auto-terms now depend on tτ instead of τ alone in the constant speed case.

The two cross-terms are each a product of three terms:

- ✓ The first one depends only on time and follows the variations of $\alpha_0(t) = f_1(t) - f_2(t)$,

- ✓ The second one depends only on τ,
- ✓ The third one depends on the product $t\,\tau$, as in the auto-terms.

It should be possible to detect the first term by applying a "Speed correlation", though, due to the third term, it should not exhibit such a sharp "order line" as the cyclic spectral line observed in the stationary case.

2.3 Speed Transform based diagnosis tools

Speed transform was first introduced in [1]. It consists in decomposing a signal over a basis of oscillatory functions $b_n(t)$ whose frequencies follow the speed variations. It consists in calculating $\frac{1}{T}\int_0^T s(t)b_0^*(t)dt$ for all the basis functions:

$$b_o(t) = e^{2\pi j\, o \int_0^t f_r(u)du} \tag{7}$$

Where $f_r(t)$ denotes the rotation frequency and o the order. More details about the accuracy of the transform and its asymptotic properties can be found in [1]. This basis was proved to be asymptotically orthonormal in the case of linear speed variations. Speed transform exhibits speed lines at all orders corresponding to oscillatory components whose frequencies are proportional to $f_r(t)$. It is an efficient tool for the estimation of the amplitude of components whose frequencies follow the speed variations. We propose to extend two signal processing tools classically used for roller bearing diagnosis in constant speed to the case of linear speed variations by replacing Fourier Transform (FT) by Speed Transform (ST).

By replacing FT by ST in the envelope spectrum, we obtain an Envelope Speed Transform (EST), calculated through the following steps:

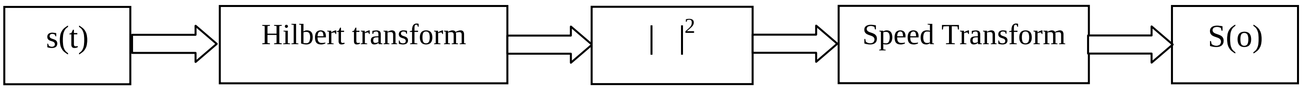

Figure 1: Synoptic of the Envelope Speed Transform calculation, where o stands for the order relatively to the rotation frequency

Spectral correlation function is a Fourier transform of the autocorrelation function both over time and time-lag. Fourier transform over time gives cyclic frequency while Fourier transform over time-lag gives spectral frequency. When applied to cyclostationary signals, spectral correlation exhibits spectral lines in cyclic frequency. We thus replace Fourier Transform by Speed Transform over time, while we still apply Fourier transform over time-lag. The function obtained, that will be called in what follows Speed Correlation, thus depends on spectral frequency and order instead of spectral frequency and cyclic frequency. In the presence of components whose frequencies follow the speed variations, it should exhibit "order lines" versus order.

2.4 Application of the proposed tools to the simplified signal

The simulated signal generated here can be described by Eq. 3 with the following parameters:

- ✓ Sampling frequency: $f_e = 100$ kHz
- ✓ Number of samples: 20000, which corresponds to 0.2 seconds
- ✓ Rotation frequency: $f_r(t) = 500 + 2500 * t$
- ✓ Frequency of the first component: $f_1(t) = 2 * f_r(t)$

- ✓ Frequency of the second component: $f_2(t) = 3 * f_r(t)$
- ✓ The amplitudes a_1=a_2 are complex, random, Gaussian and totally correlated.

The tachometer signal is a cosine wave at the rotation frequency. Speed envelope spectrum is applied to the whole length of the signal, with Hamming windowing, and interpolated by 2. The EST of the simplified signal is displayed on Fig. 2. As expected from the theoretical study, an order line appears at order K such that $f_1(t) - f_2(t) = K\, f_r(t)$, i.e. K=1.

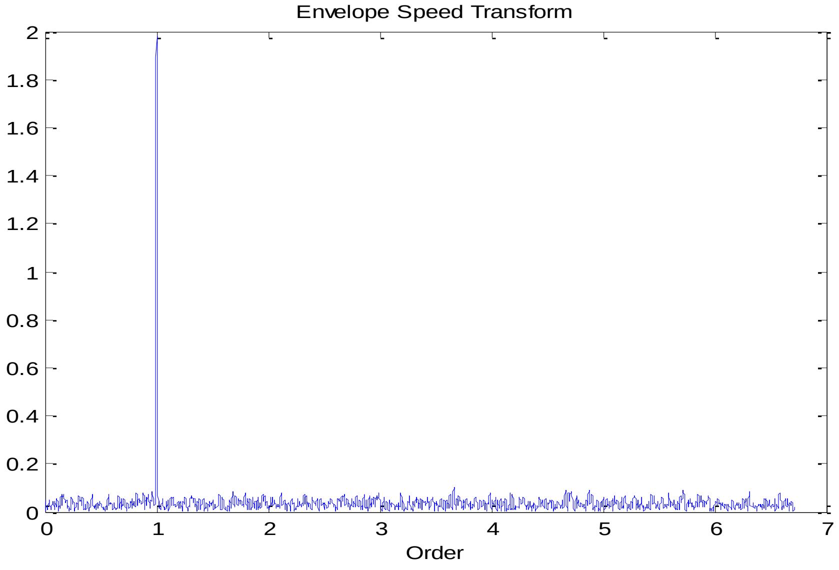

Figure 2: Envelope Speed Transform of the simplified signal

The Speed Correlation of the same signal was calculated for an order range going from 0.95 to 1.05 by steps of 0.001. It was estimated by averaged periodogram over 100 slices with 2/3 overlap. As can be observed on the result displayed on Fig. 3, an order line appears at order one, in spite of the weighing term that depends on $t\tau$.

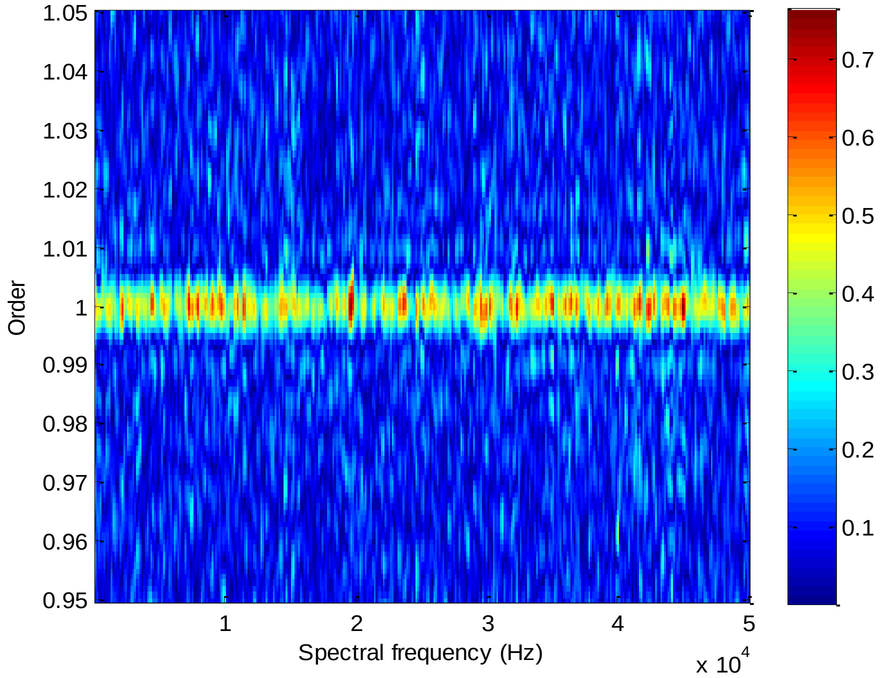

Figure 3: Speed correlation of the simplified signal

3. Application of the Speed Transform to roller bearing signals

3.1 Application to a simulated roller bearing vibration signal

Once validated on the simplified signal the two proposed techniques were applied to a simulated roller bearing signal following Eq. 1. The parameters of the model are the following ones:

- ✓ Mean diameter: $D_m = 39mm$
- ✓ Ball diameter: $d = 7.5mm$
- ✓ Number of rolling elements: $Z = 13$
- ✓ Contact angle: $\alpha = 0°$
- ✓ Resonance frequency: $f_{res} = 8\ kHz$
- ✓ Rotation frequency: $f_r = 50 + 3.33\ t$
- ✓ Amplitude modulation: $m(t) = 1 + cos\left(2\pi \int_0^t f_r(u)du\right)$
- ✓ Outer ring damage of frequency: $f_d(t) = 5.25\ f_r(t)$
- ✓ 7% random jitter on the damage period
- ✓ Signal duration: 2 seconds
- ✓ Sampling frequency: 100 kHz
- ✓ Additive Gaussian random noise. The Signal to Noise Ratio is $SNR = 7dB$

The envelope Speed Transform of this simulated vibration signal is given in Fig. 4. It exhibits a classical envelope spectrum pattern, with a speed line at order 5.25 and sidebands due to the modulation at the rotation frequency.

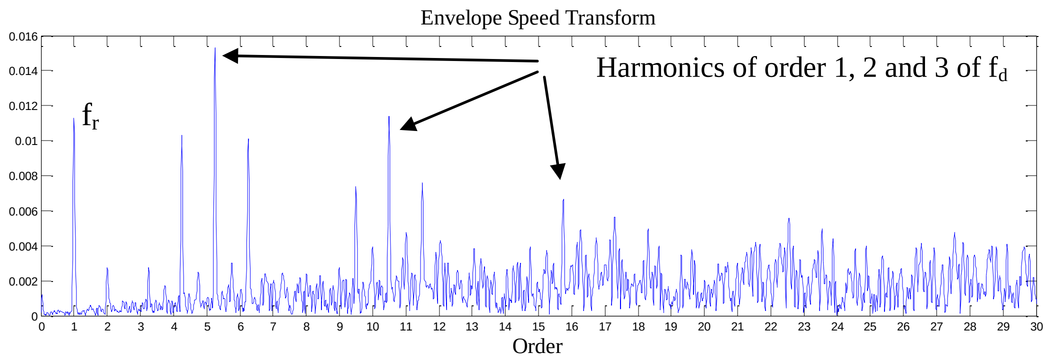

Figure 4: Envelope Speed Transform of the simulated roller bearing vibration signal

The speed correlation was computed on the same simulated vibration signal for orders ranging from 4 to 6.5 by step of 0.001. It was estimated by averaged periodogram over 100 slices with 2/3 overlap. It is normalized by the energy of the spectrum, so that the function displayed in Fig. 5 corresponds to Speed coherence (equivalent to spectral coherence if computed with Fourier transform). It exhibits a pattern that is characteristic of the chosen damage, with a peak of energy at order 5.25 and sidebands due to the rotation frequency. These peaks are wider than actual lines, which is probably due to the factor varying in $t\tau$.

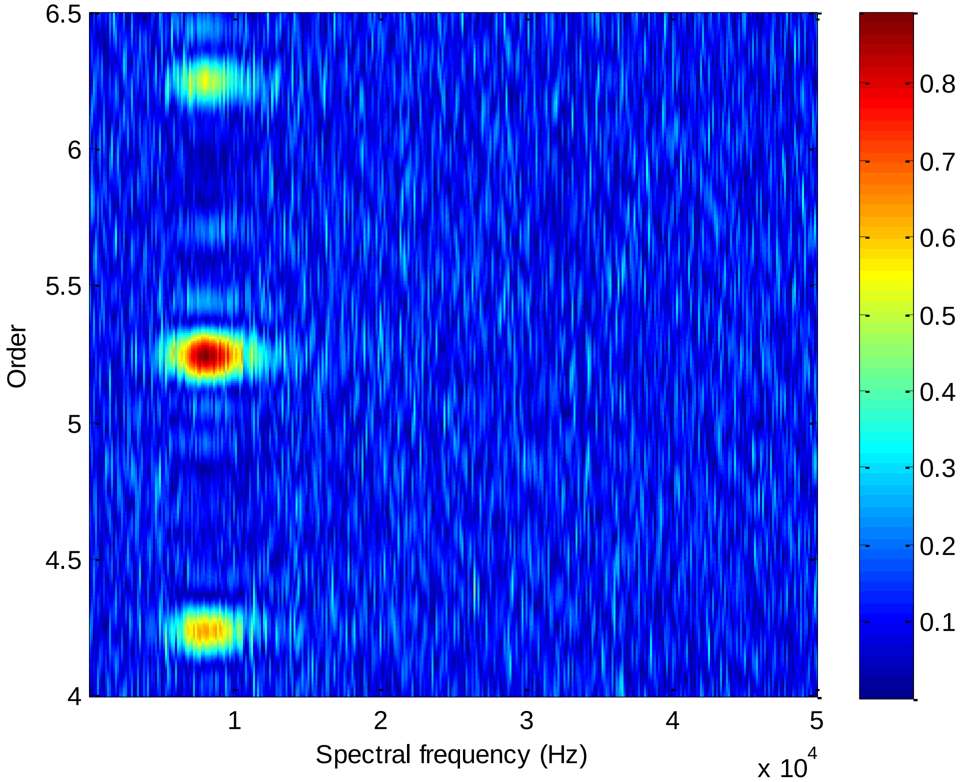

Figure 5: Speed coherence of the simulated roller bearing vibration signal

3.2 Application to a real-life roller bearing vibration signal

The signal was recorded with an accelerometer on a Spetraquest test bench. The roller bearing characteristics are the following ones:

- ✓ Number of balls: $Z = 8$
- ✓ Ball diameter: $d = .3125$ *inches*
- ✓ Mean diameter: $D_m = 1.319$ *inches*
- ✓ The sampling frequency is $f_s = 51.2$ *kHz*
- ✓ Damage on the outer ring

From these characteristic, we can deduce that the damage frequency should be equal to $f_d = 3.05 * f_r$ with f_r the rotation frequency, so that the envelope Speed Transform should exhibit lines at order $o_d = 3.05$ and its harmonic orders.

The rotation speed, estimated from a tachometer signal, is plotted on Fig. 6.

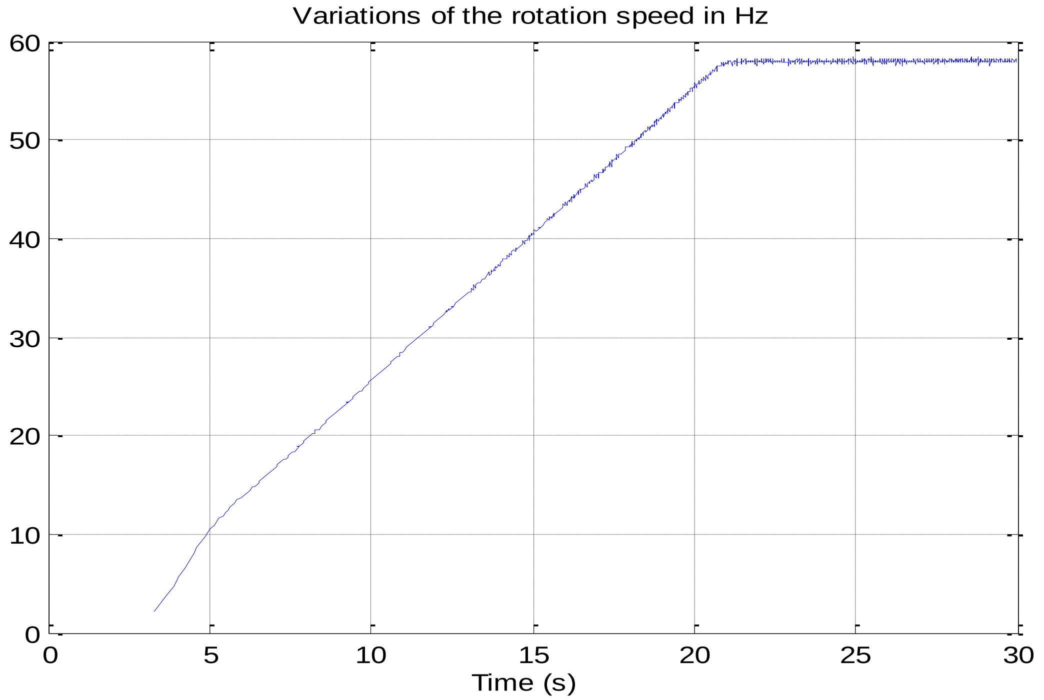

Figure 6: Variations of the rotation frequency

Both envelope spectrum and envelope Speed Transform were computed over a part of the signal taken between 8s and 16.5s and beforehand high pass filtered at f_s / 4. The two transforms are plotted on Fig. 7. On the chosen time interval, the mean rotation frequency is 32.5 Hz. The envelope spectrum is displayed from 0 Hz to 1300 Hz in order to take into account the harmonics of that mean frequency up to the 40th. Almost nothing can be detected from the classical envelope spectrum, whereas the envelope Speed Transform exhibits order lines at order $O_d = 3.025$, which is very close to the theoretical order $o_d = 3.05$ and its harmonic orders.

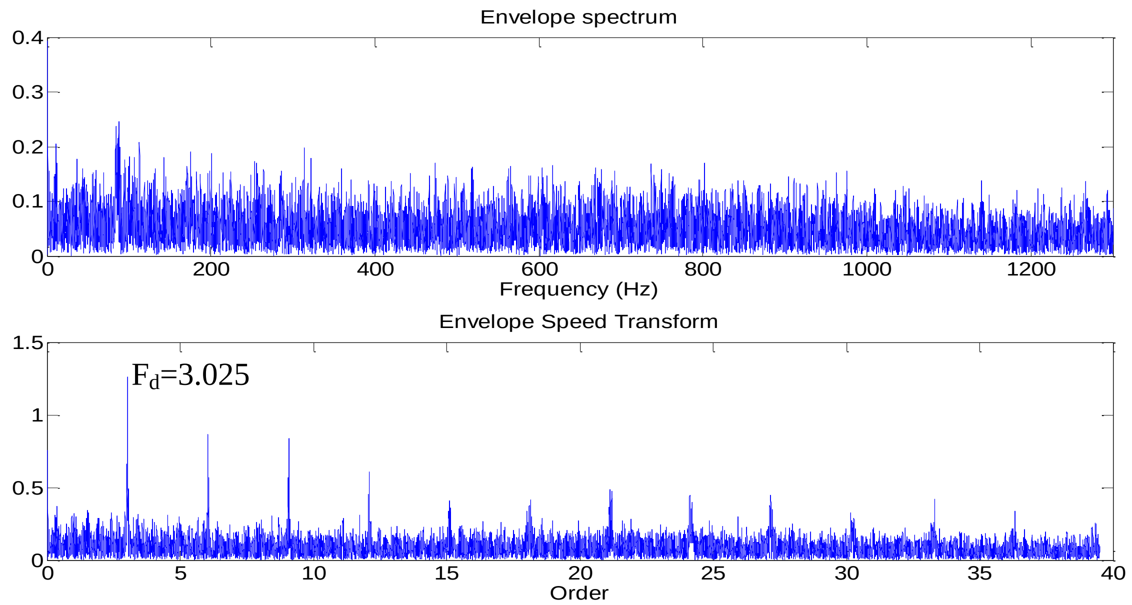

Figure 7: Envelope spectrum and envelope Speed Transform of the real-life vibration signal.

4. Conclusion

Here, some classical tools of roller bearing vibrations analysis have been extended to time varying operation condition by replacing Fourier transform by Speed transform. The relevance of such techniques have been shown by studying the effect of speed variations on a simplified model of these vibrations. We proposed two new analysis tools, called envelope speed transform and speed correlation. Envelope Speed Transform seems to be a very promising diagnosis tools both because it is very well fitted to the time varying model of the vibrations, and because it is applied over the whole length of the signal, which ensures the orthonormality of speed transform. Speed correlation also gave interesting results on the simulated signals, though it does not appear from the theoretical study as well fitted to the model as envelope Speed Transform, and the application to short slices of the signal can deteriorate the orthonormality. The theoretical aspects of Speed Transform and its extension to the case of non linear variations are still under study.

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A Autocorrelation function of the simplified model in variable speed operation

$$s(t) = a_1 e^{2\pi j \int_0^t f_1(u)du} + a_2 e^{2\pi j \int_0^t f_2(u)du}$$

Amplitudes a_1 and a_2 are complex, random and can be either correlated or not.

A.1 General case

$$R_s(t,\tau) = E\{s(t)s^*(t-\tau)\}$$

$$R_s(t,\tau) = E\left\{\left(a_1 e^{2\pi j \int_0^t f_1(u)du} + a_2 e^{2\pi j \int_0^t f_2(u)du}\right)\left(a_1 e^{2\pi j \int_0^{t-\tau} f_1(u)du} + a_2 e^{2\pi j \int_0^{t-\tau} f_2(u)du}\right)^*\right\}$$

$$R_s(t,\tau) = R_1(t,\tau) + R_2(t,\tau) + R_{2,1}(t,\tau) + R_{1,2}(t,\tau)$$

$$R_1(t,\tau) = E\{|a_1|^2\} e^{2\pi j \left(\int_0^t f_1(u)du - \int_0^{t-\tau} f_1(u)du\right)} = E\{|a_1|^2\} e^{2\pi j \int_{t-\tau}^t f_1(u)du}$$
$$R_2(t,\tau) = E\{|a_2|^2\} e^{2\pi j \left(\int_0^t f_2(u)du - \int_0^{t-\tau} f_2(u)du\right)} = E\{|a_2|^2\} e^{2\pi j \int_{t-\tau}^t f_2(u)du}$$

$$R_{2,1}(t,\tau) = E\{a_2 a_1^*\} e^{2\pi j \left(\int_0^t f_2(u)du - \int_0^{t-\tau} f_1(u)du\right)}$$
$$R_{1,2}(t,\tau) = E\{a_1 a_2^*\} e^{2\pi j \left(\int_0^t f_1(u)du - \int_0^{t-\tau} f_2(u)du\right)}$$

A.2 Peculiar case of constant frequencies

$$R_1(t,\tau) = E\{|a_1|^2\}e^{2\pi j f_1 \tau}$$
$$R_2(t,\tau) = E\{|a_2|^2\}e^{2\pi j f_2 \tau}$$

$$R_{2,1}(t,\tau) = E\{a_2 a_1^*\}e^{2\pi j \left((f_2-f_1)t+f_1\tau\right)}$$
$$R_{1,2}(t,\tau) = E\{a_1 a_2^*\}e^{2\pi j \left((f_1-f_2)t+f_2\tau\right)}$$

The two components $R_1(t,\tau)$ and $R_2(t,\tau)$ depend only on τ. The two cross terms depend both on t and τ. These terms are periodic versus time with t at frequency $f_1 - f_2$, so that $s(t)$ is cyclostationary at that frequency.

A.3 Peculiar case of uncorrelated amplitudes

In this case the cross-terms $R_{2,1}(t,\tau)$ and $R_{1,2}(t,\tau)$ are equal to zero. The autocorrelation function thus does not depend on time and $s(t)$ is stationary.

A.4 Case of the time varying frequencies

Let us suppose that the frequencies $f_1(t)$ and $f_2(t)$ are linearly varying.

$$f_1 = \alpha_1 t + \beta_1$$
$$f_2 = \alpha_2 t + \beta_2$$

$$R_1(t,\tau) = E\{|a_1|^2\}e^{2\pi j \left(\alpha_1 t\tau + \beta_1\tau - \frac{\alpha_1}{2}\tau^2\right)}$$
$$R_2(t,\tau) = E\{|a_2|^2\}e^{2\pi j \left(\alpha_2 t\tau + \beta_2\tau - \frac{\alpha_2}{2}\tau^2\right)}$$

$$R_{2,1}(t,\tau) = E\{a_2 a_1^*\}\, e^{2\pi j \left(\frac{(\alpha_2-\alpha_1)}{2}t^2 + (\beta_2-\beta_1)t\right)}\, e^{2\pi j \left(-\frac{\alpha_1}{2}\tau^2 + \beta_1\tau\right)}\, e^{2\pi j\, \alpha_1 t\tau}$$
$$R_{1,2}(t,\tau) = E\{a_1 a_2^*\}\, e^{2\pi j \left(\frac{(\alpha_1-\alpha_2)}{2}t^2 + (\beta_1-\beta_2)t\right)}\, e^{2\pi j \left(-\frac{\alpha_2}{2}\tau^2 + \beta_2\tau\right)}\, e^{2\pi j\, \alpha_2 t\tau}$$

A.5 Peculiar case $\tau = 0$

$$R_1(t,0) = E\{|a_1|^2\}$$
$$R_2(t,0) = E\{|a_2|^2\}$$

$$R_{2,1}(t,\tau) = E\{a_2 a_1^*\}\, e^{2\pi j \left(\frac{(\alpha_2-\alpha_1)}{2}t^2 + (\beta_2-\beta_1)t\right)}$$
$$R_{1,2}(t,\tau) = E\{a_1 a_2^*\}\, e^{2\pi j \left(\frac{(\alpha_1-\alpha_2)}{2}t^2 + (\beta_1-\beta_2)t\right)}$$

The autocorrelation function then becomes:

$$R_s(t,0) = E\{|s(t)|^2\} = E\{|a_1|^2\} + E\{|a_2|^2\} + 2\, Re\{a_1 a_2^*\} cos\left[2\pi\left(\frac{(\alpha_2-\alpha_1)}{2}t^2 + (\beta_2-\beta_1)t\right)\right]$$

It thus depends on t and follows the variations of $f_1(t) - f_2(t)$.