

A History of Cepstrum Analysis and its Application to Mechanical Problems

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Abstract

It is not widely realised that the first paper on cepstrum analysis was published two years before the FFT algorithm, despite having Tukey as a common author, and its definition was such that it was not reversible even to the log spectrum. After publication of the FFT in 1965, the cepstrum was redefined so as to be reversible to the log spectrum, and shortly afterwards Oppenheim and Schaffer defined the “complex cepstrum”, which was reversible to the time domain. They also derived the analytical form of the complex cepstrum of a transfer function in terms of its poles and zeros. The cepstrum had been used in speech analysis for determining voice pitch (by accurately measuring the harmonic spacing), but also for separating the formants (transfer function of the vocal tract) from voiced and unvoiced sources, and this led quite early to similar applications in mechanics. The first was to gear diagnostics (Randall), where the cepstrum greatly simplified the interpretation of the sideband families associated with local faults in gears, and the second was to extraction of diesel engine cylinder pressure signals from acoustic response measurements (Lyon and Ordubadi). Later Polydoros defined the differential cepstrum, which had an analytical form similar to the impulse response function, and Gao and Randall used this and the complex cepstrum in the application of cepstrum analysis to modal analysis of mechanical structures. Antoni proposed the mean differential cepstrum, which gave a smoothed result. The cepstrum can be applied to MIMO systems if at least one SIMO response can be separated, and a number of blind source separation techniques have been proposed for this. Most recently it has been shown that even though it is not possible to apply the complex cepstrum to stationary signals, it is possible to use the real cepstrum to edit their (log) amplitude spectrum, and combine this with the original phase to obtain edited time signals. This has already been used for a wide range of mechanical applications.

1 Introduction

The first paper on cepstrum analysis [1] defined it as “the power spectrum of the logarithm of the power spectrum”. The original application was to the detection of echoes in seismic signals, where it was shown to be greatly superior to the autocorrelation function, because it was insensitive to the colour of the signal. This purely diagnostic application did not require returning to the log spectrum, and the reason for the definition was presumably that software for producing power spectra was readily available. Even though [1] had one of the authors of the FFT algorithm (John Tukey) as a co-author it was published two years before the FFT algorithm [2] and thus the potential of the latter was not yet realised. Tukey himself in [1] writes “Although spectral techniques, involving second-degree operations, are now quite familiar, the apparently simpler first-degree Fourier techniques are less well known” (he then refers to a paper of which he is the sole author, which confirms that he made the remark). Thus, the original cepstrum was not reversible even to the log spectrum, and so even though the word and concept of a “lifter” (a filter in the cepstrum) was defined in the original paper, this was enacted as a convolutive filter applied to the log spectrum rather than a window in the cepstrum. The diagnostic application was also satisfactory for determining the voice pitch of voiced speech [3], so speech analysis was one of the earliest applications, much of the development being done at Bell Telephone Labs.

The early history of the development of the cepstrum was recently published by the two pioneers Oppenheim and Schaffer [4], but their paper primarily covers applications in speech analysis,

communications, seismology and geophysics, and no mechanical applications are given. They describe how Oppenheim's PhD dissertation at MIT [5] introduced the concept of "homomorphic systems", where nonlinear relationships could be converted into linear ones to allow linear filtering in the transform domain. Typical examples were conversion of multiplication into addition by the log operation, and conversion of convolution by first applying the Fourier transform (to convert the convolution to a multiplication) followed by the logarithmic conversion. It was a small further step to perform a (linear) inverse Fourier transform on the log spectrum to obtain a new type of cepstrum, and this was the basis of Schafer's PhD dissertation at MIT [6]. By retaining the phase in all operations, the "complex cepstrum" was defined as the inverse Fourier transform of the complex logarithm of the complex spectrum, and this was thus reversible to the time domain. Oppenheim and Schafer collaborated on a book [7], in which a number of further developments and applications of the cepstrum were reported, in particular the analytical form of the cepstrum for sampled signals, where the Fourier transform could be replaced by the z-transform.

2 Basic Relationships and Definitions

2.1 Formulations

The original definition of the (power) cepstrum was:

$$C_p(\tau) = \left| \mathfrak{F} \{ \log(F_{xx}(f)) \} \right|^2 \quad (1)$$

where $F_{xx}(f)$ is a power spectrum, which can be an averaged power spectrum or the amplitude squared spectrum of a single record.

The definition of the complex cepstrum is:

$$C_c(\tau) = \mathfrak{F}^{-1} \{ \log(F(f)) \} = \mathfrak{F}^{-1} \{ \ln(A(f)) + j\phi(f) \} \quad (2)$$

where

$$F(f) = \mathfrak{F} \{ f(t) \} = A(f)e^{j\phi(f)} \quad (3)$$

in terms of the amplitude and phase of the spectrum. It is worth noting that the "complex cepstrum" is real, despite its name, because the log amplitude of the spectrum is even, and the phase spectrum is odd.

The new power cepstrum is given by:

$$C_p(\tau) = \mathfrak{F}^{-1} \{ \log(F_{xx}(f)) \} \quad (4)$$

which for the spectrum of a single record (as in (3)) can be expressed as:

$$C_p(\tau) = \mathfrak{F}^{-1} \{ 2\ln(A(f)) \} \quad (5)$$

The so-called real cepstrum is obtained by setting the phase to zero in Eq. (2):

$$C_r(\tau) = \mathfrak{F}^{-1} \{ \ln(A(f)) \} \quad (6)$$

which is seen to be simply a scaled version of (5).

It should be noted that the complex cepstrum requires the phase $\phi(f)$ to be unwrapped to a continuous function of frequency, and this places a limit on its application. It cannot be used for example with stationary signals, made up of discrete frequency components (where the phase is undefined at intermediate frequencies) and stationary random components (where the phase is random). This point is taken up later. It can only be used with well-behaved functions such as impulse responses, where the phase is well-defined (and often related to the log amplitude). The unwrapping can usually be accomplished using an algorithm such as UNWRAP in Matlab®, although the latter may give errors where the slope of the phase function is steep. This problem can usually be resolved by repeatedly interpolating the spectra by factors of 2 until the same

result is obtained with successive steps. The penultimate step can then be used. Errors in unwrapping can also occur when the amplitude is very small, so that noise affects the phase estimates.

As mentioned above, Oppenheim and Schaffer in [7] derived the analytical formulation of the cepstrum for transfer functions, from digitised signals, expressed using the z-transform in terms of their poles and zeros in the z-plane. Thus, for a general impulse response function:

$$h(n) = \sum_{i=1}^{2N} A_i e^{s_i n \Delta t} = \sum_{i=1}^{2N} A_i z_i^n \quad (7)$$

where the A_i are residues, s_i and z_i poles in the s-plane and z-plane, respectively, and Δt the time sample spacing, the transfer function in the z-plane is given by:

$$H(z) = \frac{B \prod_{i=1}^{M_i} (1 - a_i z^{-1}) \prod_{i=1}^{M_o} (1 - b_i z)}{\prod_{i=1}^{N_i} (1 - c_i z^{-1}) \prod_{i=1}^{N_o} (1 - d_i z)} \quad (8)$$

where the c_i and a_i are poles and zeros inside the unit circle in the z-plane, respectively, and the d_i and b_i are the (reciprocals of the) poles and zeros outside the unit circle, respectively (so that $|a_i|, |b_i|, |c_i|, |d_i| < 1$). Oppenheim and Schaffer [7] show that for this transfer function $H(z)$, the cepstrum can be represented as:

$$\begin{aligned} C_h(n) &= \ln(B) & , n = 0 \\ C_h(n) &= -\sum_i \frac{a_i^n}{n} + \sum_i \frac{c_i^n}{n} & , n > 0 \\ C_h(n) &= \sum_i \frac{b_i^{-n}}{n} - \sum_i \frac{d_i^{-n}}{n} & , n < 0 \end{aligned} \quad (9)$$

Here, n is time sample number, known as ‘‘quefrequency’’ (see Section 2.2). It will be seen that the poles and zeros inside the unit circle, which define the minimum phase part of the transfer function, have a causal cepstrum, with positive quefrequency components only, while the poles and zeros outside the unit circle, defining the maximum phase part, form the cepstrum at negative quefrequency. Oppenheim and Schaffer thus were able to demonstrate that the real and imaginary parts of the Fourier transform of the cepstrum of a minimum phase function (the log amplitude and phase spectra, respectively) are Hilbert transforms of each other. Hence, if it is known that a transfer function is minimum phase (often the case for many simple mechanical structures), it is not necessary to measure or unwrap the phase, as it can be calculated from the log amplitude. In fact, the easiest way to do this is to form the complex cepstrum (one-sided) from the real cepstrum (real and even) by doubling positive quefrequency components and setting negative quefrequency components to zero. The log amplitude and phase spectra can then be obtained by a forward transform of the complex cepstrum.

One further property of the cepstrum, already mentioned as a homomorphic property, was developed quite early, and that is that forcing function and transfer function effects are additive in the cepstrum. For SIMO (single input, multiple output) systems, any output signal is the convolution of the input signal with the impulse response function of the transmission path, and this convolutive relationship is converted to an addition by the cepstrum operation as follows:

$$\text{If} \quad y(t) = f(t) * h(t) \quad (10)$$

$$\text{then} \quad Y(f) = F(f) \cdot H(f) \quad (11)$$

$$\log(Y(f)) = \log(F(f)) + \log(H(f)) \quad (12)$$

and
$$\mathfrak{T}^{-1}\{\log(Y(f))\} = \mathfrak{T}^{-1}\{\log(F(f))\} + \mathfrak{T}^{-1}\{\log(H(f))\} \quad (13)$$

Note that this only applies for SIMO systems, since for MIMO (multiple input, multiple output) systems each response signal is a sum of convolutions. This point is taken up later.

This relationship was used in speech analysis, for example to separate the speech source (voiced or unvoiced) from the transmission effect of the vocal tract (whose resonances are called “formants”), and in [7] a vocoder is proposed where the speech information can be considerably compressed by transmitting only the low quefrequency part of the cepstrum (giving the formants) and an indicator of whether the speech segment is voiced or unvoiced, and if so the voice pitch. To reconstruct the speech, the formants were used to generate a filter, which was excited either by a noise source (unvoiced) or a periodic pulse generator with frequency the same as the voice pitch.

2.2 Terminology

In the original paper [1], the authors coined the word “cepstrum” by reversing the first syllable of “spectrum”, the justification being that it was a “spectrum of a spectrum”. Similarly, the word “quefrequency” was obtained from “frequency”, and the authors also suggested a number of others, including:

rahmonic	from	harmonic
lifter	from	filter
gamnitude	from	magnitude
saphe	from	phase
darius	from	radius
dedomulation	from	demodulation

Of these, cepstrum, quefrequency, rahmonic and lifter are useful in clarifying that the operations or features refer to the cepstrum, rather than the spectrum or time signal, and are still regularly used in the literature, as well as in this paper. Note that the autocorrelation function is also a “spectrum of a spectrum”, so the distinctive feature of the cepstrum is the log conversion of the spectrum.

3 Early Applications in Mechanics and Acoustics

3.1 Diagnostics of families of harmonics or sidebands

The author became aware of the possibility of using the cepstrum to detect and quantify families of periodically spaced spectral components by reading [3] and realised that this would not only detect families of harmonics, but also equally spaced modulation sidebands. Local faults in gears give an impulsive modulation of the gearmesh signals (both amplitude and frequency modulation) resulting in large numbers of sidebands spaced at the speed of the gear on which the local fault is located. The majority of such sidebands are only visible on a spectrum with a log amplitude scale, and so the cepstrum is an ideal way to collect the (average) information of a large number of such sidebands into a relatively small number of rahmonics in the cepstrum, of which the first contains most of the information. He had even experimented with calculating the cepstrum using FFT methods before joining the company Brüel and Kjær in Copenhagen in 1971. A Fortran version of the FFT algorithm was obtained from [8]. However, even though B&K had a fully designed mini-computer-based FFT analyser in 1971, this was never released onto the market, and so other methods of obtaining the cepstrum were sought. In 1973 an Application Note No. 13-150 entitled “Cepstrum Analysis and Gearbox Fault Diagnosis” [9] was published by B&K, in which six methods were described for calculating the (power) cepstrum, primarily for the purpose of diagnosing gear faults. Several were variations on the theme of using a heterodyne analyser for performing the “frequency” analysis on a linear frequency scale with constant bandwidth, first to obtain the original power spectrum on a log amplitude scale, and then the second analysis of the log spectrum to the cepstrum on a linear scale. In between, the log spectrum was stored in a medium, which could either be a circulating digital memory (a Digital Event Recorder type 7502), or a tape loop on an FM tape recorder. In both cases the stored spectrum was played back at a much higher rate than recorded. The initial analysis could either be generated directly from a continuous recording of the signal on a tape recorder, or sometimes from a recirculated section of signal recorded in a second Digital

Event Recorder, possibly using an analogue “Gauss Impulse Multiplier” type 5623, to apply a Gaussian, rather than Hanning, weighting to the selected and periodically repeated signal section.

A result using a Digital Event Recorder for intermediate storage is shown in Figure 1, with both frequency analyses done by a heterodyne analyser. In the log spectrum on the left, a large number of sidebands are visible around the first three harmonics of gearmesh frequency, and the cepstrum on the right shows that these are spaced at the high speed shaft speed of 85 Hz. The machine analysed was a gearbox between a gas turbine at 85 Hz and a generator at 50 Hz, and the cepstrum shows immediately that the fault is on the gear rotating at turbine speed. The component was much smaller after repair. It should be noted that the frequency estimation (actually the reciprocal of the equivalent quefrequency) is very accurate because it is the average of the sideband spacing over the whole spectrum.

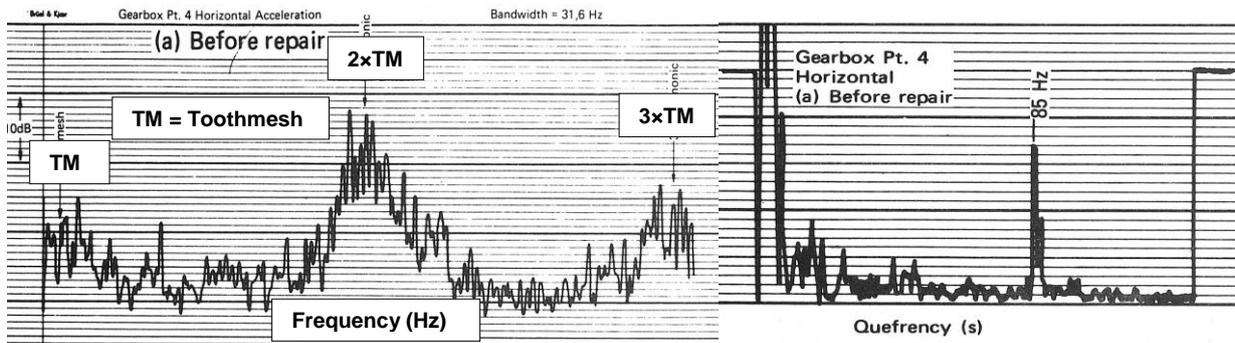


Figure 1: Log spectrum and cepstrum for a gear fault

Even though the B&K FFT analyser was not released, the designer, Arne Grøndahl, wrote driver routines to allow the assembler language FFT algorithm to be called from a Fortran program, and it was possible to perform cepstrum calculations, even the complex cepstrum, with fully FFT-based methods, as early as 1975, and the author published further results of gear diagnostics at that time [10].

At about the same time, Georges Sapy, of Electricité de France (EDF) found that the cepstrum could be used to detect damaged or missing blades in a steam turbine [11]. The lack of guidance of the steam by the missing or faulty blades gave rise to a series of impulsive interactions with each stator blade, so that a casing mounted accelerometer would detect the pulses from the nearest section at a rate corresponding to the rotor speed, 50 Hz. This then gave rise to an increase in intermediate frequency harmonics in comparison to the normal condition, as shown in Figure 2.

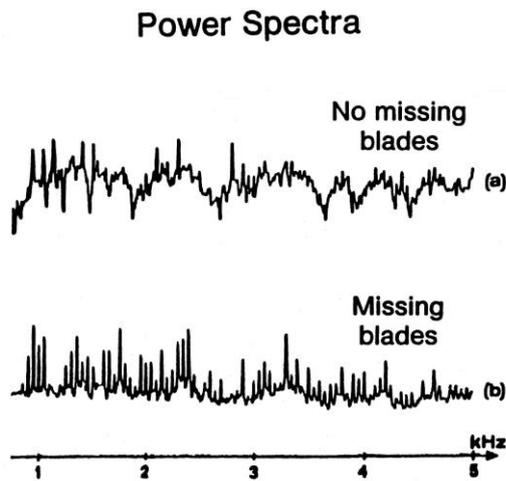


Figure 2: Log spectra (a) without blade fault (b) with blade fault

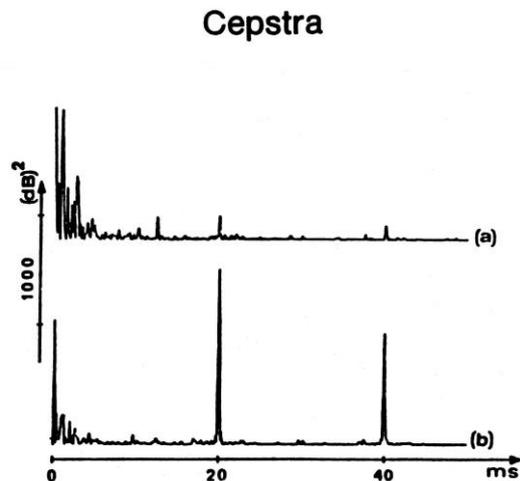


Figure 3: Cepstra (a) without blade fault (b) with blade fault

Figure 3 shows that the power cepstrum (original definition) is an efficient way of detecting and quantifying the pattern of additional harmonics in the intermediate frequency range (750-5,000 Hz).

3.2 Separation of source and transmission path

One of the first groups to apply the cepstrum to the separation of forcing function and transfer function in mechanical systems was Professor Richard Lyon of MIT, and co-workers. With his doctoral student Afarin Ordubadi he showed how the complex cepstrum could be used to extract the cylinder pressure (source) signal from measured acoustic responses [12]. Figure 4 shows the estimated pressure signals from two cylinders of a diesel engine from this paper.

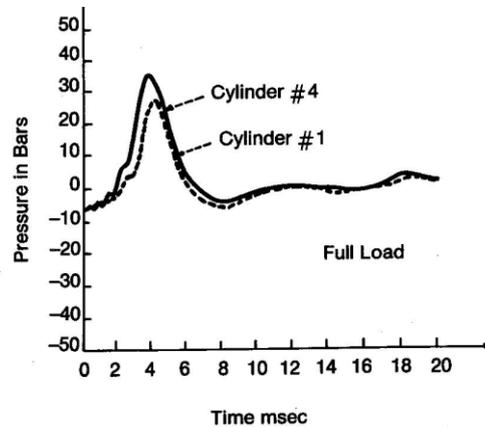


Figure 4: Estimated cylinder pressure using the complex cepstrum [12]

In 1984, the author proposed using the cepstrum for separating the gearmesh forcing function (single input) from the structural transfer function in a gearbox [13]. This was considered important because for example an increase in the second harmonic of the gearmesh frequency might be due to tooth wear (normally in two patches on each tooth, one on each side of the pitchline, where there is a rolling rather than sliding action). However, it might be because a natural frequency of the structure or internals has reduced so as to coincide with the harmonic in question because of a developing crack, and this situation might have a very different prognosis. Figure 5 shows an example [14] based on this idea, where the periodic forcing function of the gearmesh was removed in the cepstrum using a comb lifter adjusted to the gearmesh frequency, leaving a cepstrum dominated by the transfer function to the measurement point. The edited cepstrum was transformed back to the log spectrum, which shows that the latter was little changed for two measurements on the same gearbox, where one had the presence of cracked teeth. Even though a modal analysis was not carried out, it is obvious that the natural frequencies did not change appreciably.

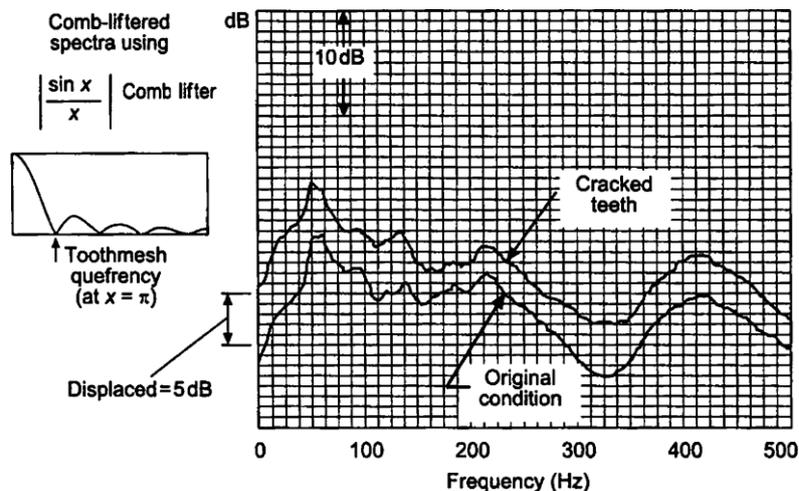


Figure 5: Use of liftering in the cepstrum to remove the forcing function **component** from the cepstra for a gear with and without cracked teeth, leaving the part dominated by the structural transfer functions [14]

4 Later Developments up to the Present

4.1 Applications of the real cepstrum in machine diagnostics

From 1975 to 1980, the author made a number of further developments to the application of the real cepstrum to gear diagnostics, presented at a number of conferences, and which culminated in the publication of a second Application Note with the same title as [9], published by B&K in 1980, but also in the journal *Maintenance Management International* [15]. In this, a number of practical considerations in the calculation and interpretation of the cepstrum are discussed, of which perhaps the most important is the recommendation to use the amplitude of the “analytic cepstrum” in place of the normal definition of the real or power cepstrum, in particular when it is based on a section of the spectrum not starting from zero frequency (eg a zoom spectrum) or when sidebands are not simultaneously a harmonic series passing through zero frequency (as can happen with planetary gears because the sun speed is not a subharmonic of the mesh frequency). In this case, the harmonics are not necessarily positive peaks, but the quefrequency corresponding to a sideband spacing can actually be at a zero crossing between a positive and negative peak. Figure 6 illustrates this for two slightly displaced zoom spectra of a gearbox signal.

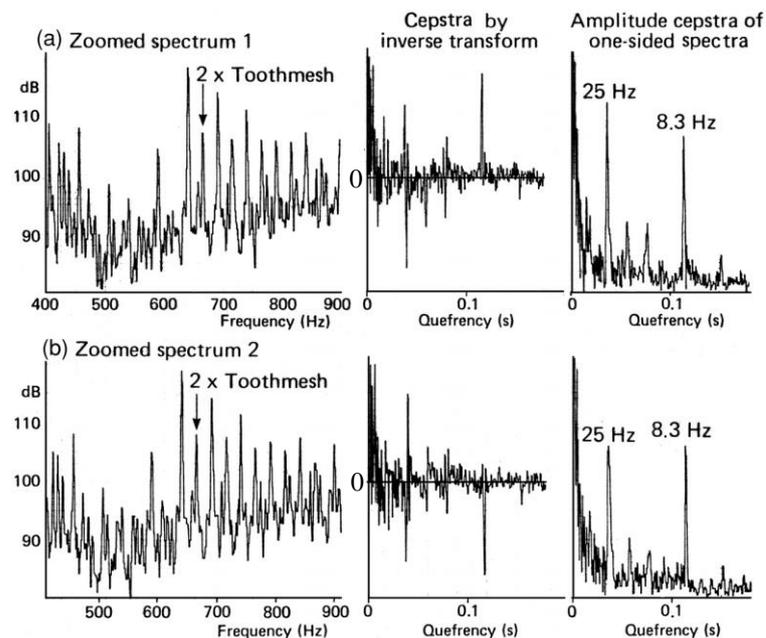


Figure 6: Advantage of the analytic cepstrum for zoom spectra

The analytic cepstrum is simply calculated using Hilbert transform theory by setting the negative frequency part of the log spectrum to zero. This means that the real and imaginary parts of the resulting analytic cepstrum are Hilbert transforms, but the magnitude or envelope always has its peak at the quefrequency corresponding to the sideband spacing. Note that the analytic cepstrum (as opposed to the “complex cepstrum”) is genuinely complex.

Ref. [16] now contains all this information plus advice on how to optimise the use of the cepstrum by editing of the log spectrum and of the cepstrum itself to modify the log spectrum.

In 1991, Capdessus and Sidahmed showed how the cepstrum was very useful for separating the effects of two gears with only a small difference in the numbers of teeth. As mentioned above, the harmonics of gearmesh frequency are surrounded by sidebands spaced at the two shaft speeds, but when the ratio is close to unity it is difficult to separate the sidebands in the spectrum. In [17] a case is presented where the numbers of teeth on the meshing gears are 20 and 21, respectively, and the two components in the cepstrum were quite distinct, allowing diagnosis of which gear developed a defect.

From 1999-2004, Mohamed El Badaoui and co-workers introduced a number of new aspects into the use of cepstrum analysis for gear diagnostics. One was presented in [18] and considerably expanded in [20], and is based on the fact that under fairly broad conditions there is a tendency for the sum of the first harmonic peaks, corresponding to each of the two gears in a meshing pair, to remain constant, so that if one increases as the result of a local fault, the other decreases correspondingly. An example from [20] is given in Fig. 7 where A1 and A2 represent the two first harmonics. The authors defined a diagnostic indicator d , which is the ratio of specially normalised versions of A1 and A2 which is relatively insensitive to signal/noise ratio, even though this changes the absolute magnitude of the numerator and denominator terms. This is seen to give a very good trend parameter for a developing fault (in fact the same data as treated in [17]).

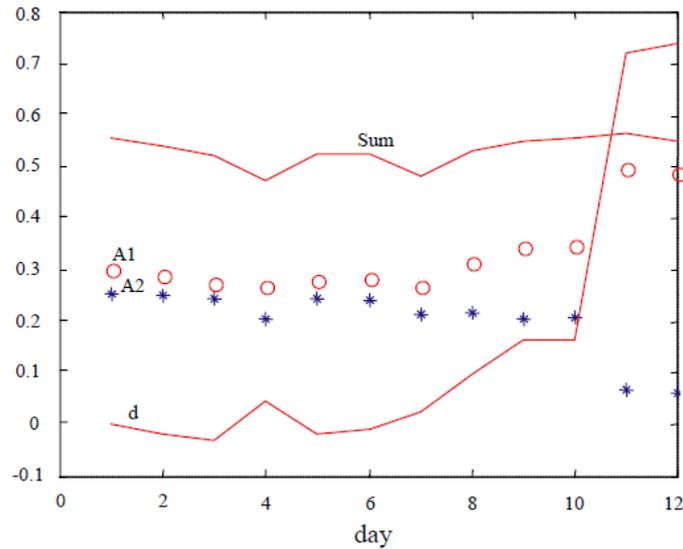


Figure 7: Application of the diagnostic indicator d from [20]

The other major contribution was based on the fact that the cepstrum of a signal with an inverted echo is entirely negative, so that its integral over quefrequency becomes markedly more negative when such an inverted echo is in the analysed section of time record. The “moving cepstrum integral” (MCI) [19] was based on this and was applied to the detection of spalled gear teeth, the rationale being that the signal on exiting a spall would be an inverted version of the signal on entry, thus giving an inverted echo. The principle is illustrated in Figure 8(a) and Fig. 8(b) shows an example of its application from [19].

Endo in [21] showed that there were inverted echoes in the signals from tooth cracks as well, so the MCI could not be used alone to distinguish them.

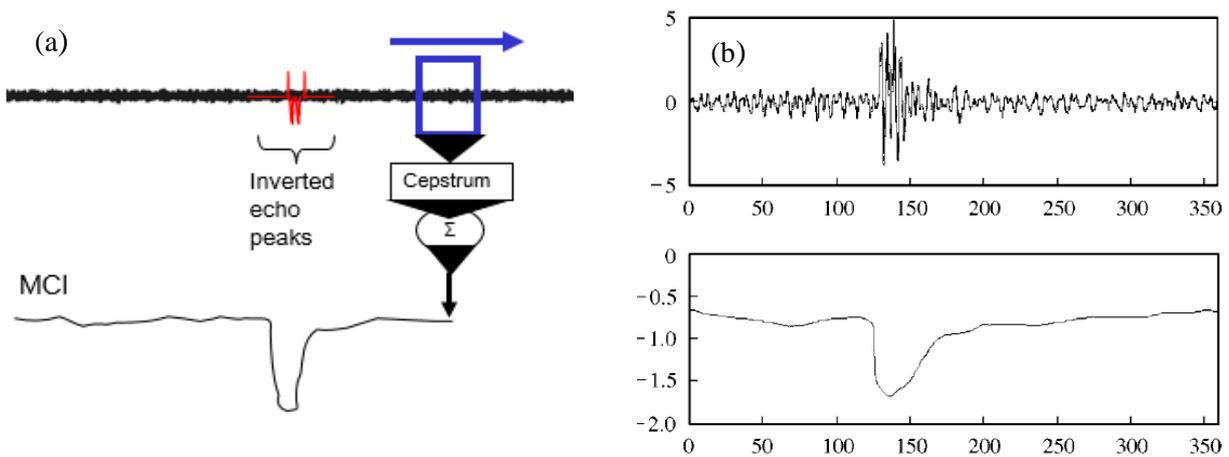


Figure 8 (a) Principle of Moving Cepstrum Integral (b) example of application from [19]

4.2 Applications of the cepstrum in modal analysis

Most of the following is discussed in more detail in [22].

4.2.1 The differential cepstrum

One further major theoretical development with the cepstrum was the proposal in 1981 by Polydoros and Fam of the differential cepstrum [23]. This was originally defined as the inverse z-transform of the derivative of the log spectrum. Thus:

$$C_d(\tau) = Z^{-1} \left\{ \frac{d}{dz} [\log H(z)] \right\} = Z^{-1} \left\{ \frac{H'(z)}{H(z)} \right\} \quad (14)$$

Because taking the derivative in one domain corresponds to multiplication by the independent variable in the other domain, the differential cepstrum can be written in terms of the poles and zeros in the z-plane (cf Equ.(9)) as:

$$\begin{aligned} C_{hd}(n) &= -\sum_i a_i^n + \sum_i c_i^n & , n > 0 \\ C_{hd}(n) &= \sum_i b_i^{-n} - \sum_i d_i^{-n} & , n < 0 \end{aligned} \quad (15)$$

Note that in [23] these equations are given for queffrequency displaced by one sample ($0 \rightarrow 1$) but the zero origin has been maintained here to emphasize the similarity with the complex cepstrum. It will be recognised that the form of the differential cepstrum (sums of complex exponentials) is the same as that of the impulse response function, except that there are similar terms for both poles and zeros (the latter with negative sign) and all “residues” are equal to unity.

4.2.2 Modal analysis using the cepstrum of response signals

In 1996, Gao and Randall [24, 25] extracted the modal properties (poles and zeros) of a transfer function from measured response signals, in a case where the forcing function was confined to the very low queffrequency region of the response cepstrum. This applies wherever the excitation is impulsive with a relatively smooth and flat spectrum over the frequency range of interest, eg a hammer blow, or in fact broadband random noise in cases where the smoothed power spectrum of the response can be used, eg for minimum phase systems where the phase does not have to be measured.

This is illustrated in Figure 9, where it can be seen that the regenerated cepstrum cannot be distinguished from the measured one in the high queffrequency region dominated by the transfer function (which was the section curve-fitted for the poles and zeros).

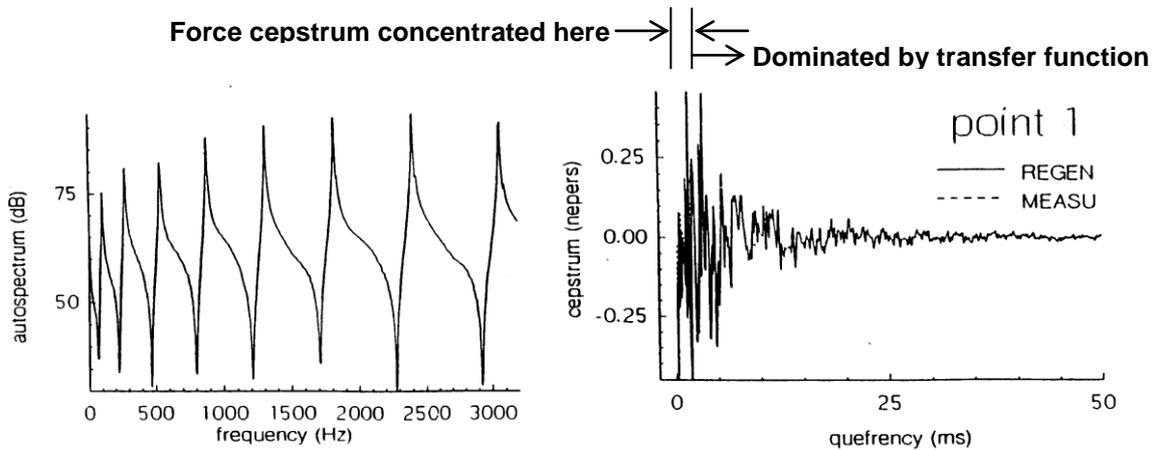


Figure 9: Extraction of the complex cepstrum of a transfer function from the response autospectrum of a beam excited by a hammer blow
(a) driving point response autospectrum (b) measured and regenerated cepstra [24]

It should be noted that the force spectrum was quite flat, but fell off by about 15 dB over the frequency range up to 3.2 kHz, though this did not affect the curve-fitting of the poles and zeros. Thus the cepstrum method does not demand that the excitation is white, as do some other methods of operational modal analysis.

Figure 9(a) shows the response autospectrum at the driving point (one end) of a free-free beam excited by a hammer blow. Noise in the response is negligible. Figure 9(b) shows the cepstrum derived directly from this, and the cepstrum of the transfer function obtained from it by curve-fitting equations of the form of (9) for the poles and zeros inside the unit circle using a least squares optimisation (Levenberg-Marquardt) method [24]. The cepstrum of the transfer function was then regenerated using Equ. (9). Minimum phase properties were assumed for this simple structure. Equally good results were obtained by curve-fitting the differential cepstrum, using both the Levenberg-Marquardt method and the ITD (Ibrahim Time Domain) method, originally developed for extracting impulse responses from free decay vibration signals.

When an attempt was made to regenerate the frequency response functions (FRFs) from the curve-fitted poles and zeros, it was realised that there are two pieces of information lacking to fully restore them:

1. An overall scaling factor that is part of the zero frequency value of the cepstrum which is not curve-fitted.
2. An equalisation curve due to the lack of residual information from out-of-band modes that are excluded from the measured cepstrum.

In [25] it is described how this information can be obtained by other means from earlier or similar FRFs, for example a finite element (FE) model of the structure. Neither of the two above factors is sensitive to small changes in the exact pole and zero positions, and so it is possible to track slow changes in the dynamic properties (such as from a developing crack) or update the FE model to have the correct (measured) in-band poles and zeros. Examples are given in [25].

In [25] the way of generating the equalisation curves was using “phantom zeros” in-band to compensate for the missing out-of-band poles and zeros. This was effectively the same as done for the established Rational Fraction Polynomial method. Even though pole-zero and pole-residue models are identical for systems with a finite number of DOFs, in practice models must be truncated. For pole-residue models, this means that the residues are correct but the zeros wrong, but for pole-zero models the zeros are correct but the residues wrong. Almost no compensation is required for driving point measurements, because there are equal numbers of poles and zeros truncated in the out-of-band region, and these tend to cancel in the in-band region. However, the fewer zeros there are in the transfer function, the greater the imbalance that has to be compensated for with the equalisation curve.

The next approach was to simply take the dB difference between the regenerated and reference FRFs, and smooth it to obtain the equalisation curve. Before smoothing, the difference curve would have peak-notches because of the mismatch in actual positions of the poles and zeros. The first attempt at smoothing was done using a lowpass lifter in the cepstrum of the difference curve, but this gave ripples because of wraparound errors in the FFT processing. Recently, much better results have been achieved by using polynomial curve-fitting for the smoothing [26], as illustrated in Figure 10. Note that the equalisation curve for the driving point measurement is almost flat, whereas that for the end-to-end measurement, with no zeros, has the steepest slope.

With respect to scaling, this method tends to give the correct result for free-free systems if the inertial properties of the rigid body test object are correct, since these determine the zero frequency values of the FRFs. This will be the case for example where the reference FRF is an actual measurement and the updating is to follow a change in stiffness (eg a developing crack). It is also relatively simple to arrange for an FE model to have the correct rigid body inertial properties. For constrained test objects, it is a little more difficult, because the zero frequency values are then determined by the stiffness properties, and these are likely to be affected by poor modelling of joints etc. However, a relatively simple updating of the FE model to match the frequencies of the first one or two elastic modes will usually ensure that the zero frequency value is correct, even if higher frequency modes are not correct because of local stiffness anomalies.

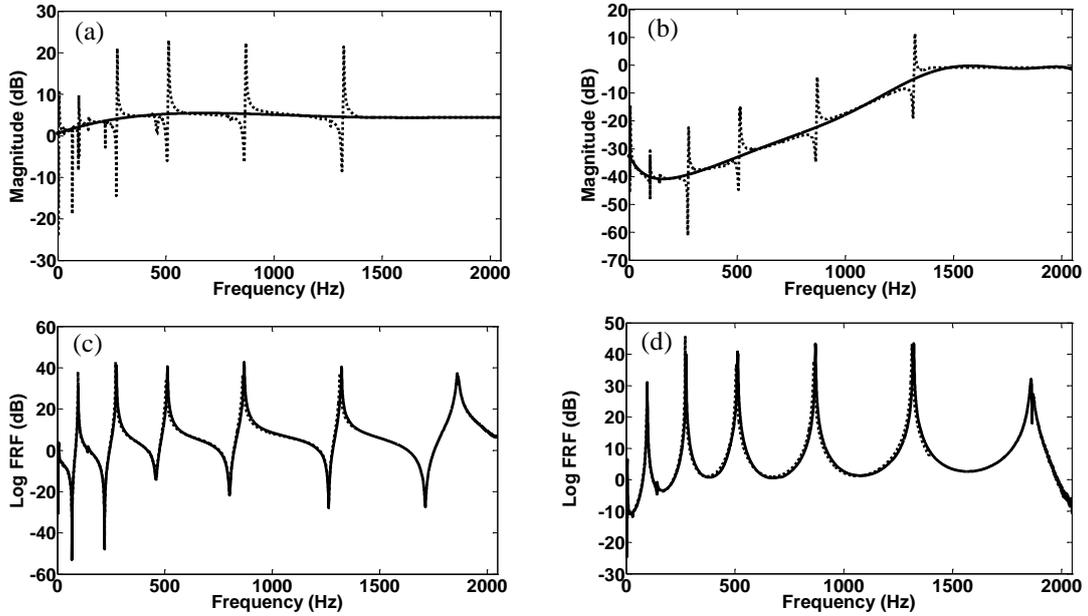


Figure 10: Driving point (left) and transfer (right) comparison showing effect of polynomial-based equalisation curves: (a, b) Equalisation curves, (c,d) FRFs compared with measurement. Measured (—), regenerated from OMA and scaled (···)

Ref. [24] also shows how the effects of echoes in a measurement (eg a double hit with a hammer) can be neutralised when curve-fitting the cepstrum because the corresponding rahmonics can be eliminated by a comb lifter.

4.2.3 The mean differential cepstrum

In 2000 [27] Antoni et al proposed the “mean differential cepstrum” (MDC), which can be calculated by a formula similar to Equ. (14), to which it reverts for a single realization:

$$C_{hmd}(\tau) = \mathfrak{F}^{-1} \left\{ \frac{E[H'(f) H^*(f)]}{E[H(f) H^*(f)]} \right\} \quad (16)$$

This permits averaging over a number of realizations, for example of the response of a system excited by a burst random sequence. Even mixed phase FRFs can be obtained using the MDC, even in the presence of extraneous noise in the signals. A typical result is shown in Figure 11.

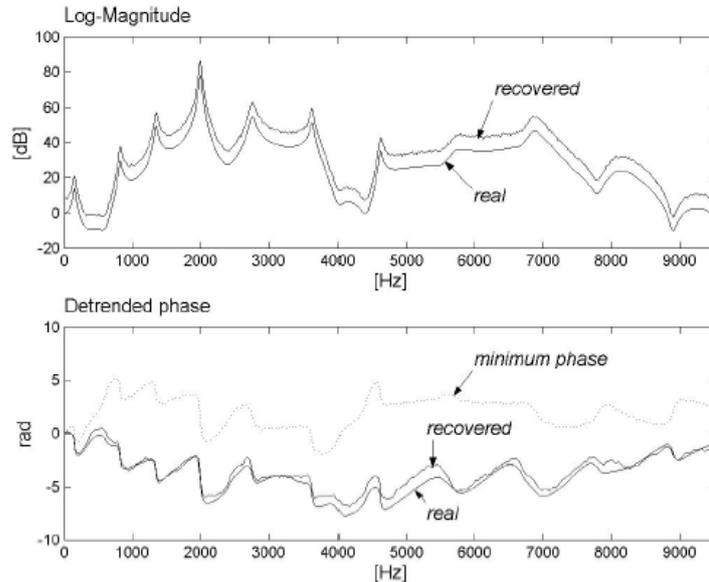


Figure 11: Non-minimum phase FRF recovered using the MDC (from [27]).

4.2.4 MIMO systems

As mentioned above, the cepstrum can only separate forcing and transfer functions for SIMO systems, so in the more general MIMO case, it is first necessary to convert it into a sum of SIMOs, or at least extract one SIMO response from the total. This is the subject of the topic “blind source separation” (BSS), and so in principle any BSS method could be used to extract responses to a single excitation. Ref. [28] is a special issue of Mechanical Systems and Signal Processing devoted to the applications of BSS to mechanical systems.

One application of particular interest, not discussed there, is the extraction of the cepstrum of the transmission path for a single cyclostationary source with a particular cyclic frequency from a MIMO response signal. Figure 12(a) shows the cyclic spectrum of the response of a rail vehicle excited by a second order cyclostationary source [29] in the presence of other excitations. In this case it was a burst random signal, but the intended application was to diesel railcars where the engine firing frequency would constitute such a source. The cyclic spectrum at cyclic frequency $\alpha = 1/T$ (with T equal to the period of the excitation frequency) is shown in the paper to contain information only of the transfer function of that source to the measurement point(s) and can thus be curve-fitted for the modal properties as though it were SIMO. Figure 12(b) shows a resulting (scaled) mode shape (OMA), and compares it with the result of an experimental modal analysis (EMA) obtained using the measured excitation force.

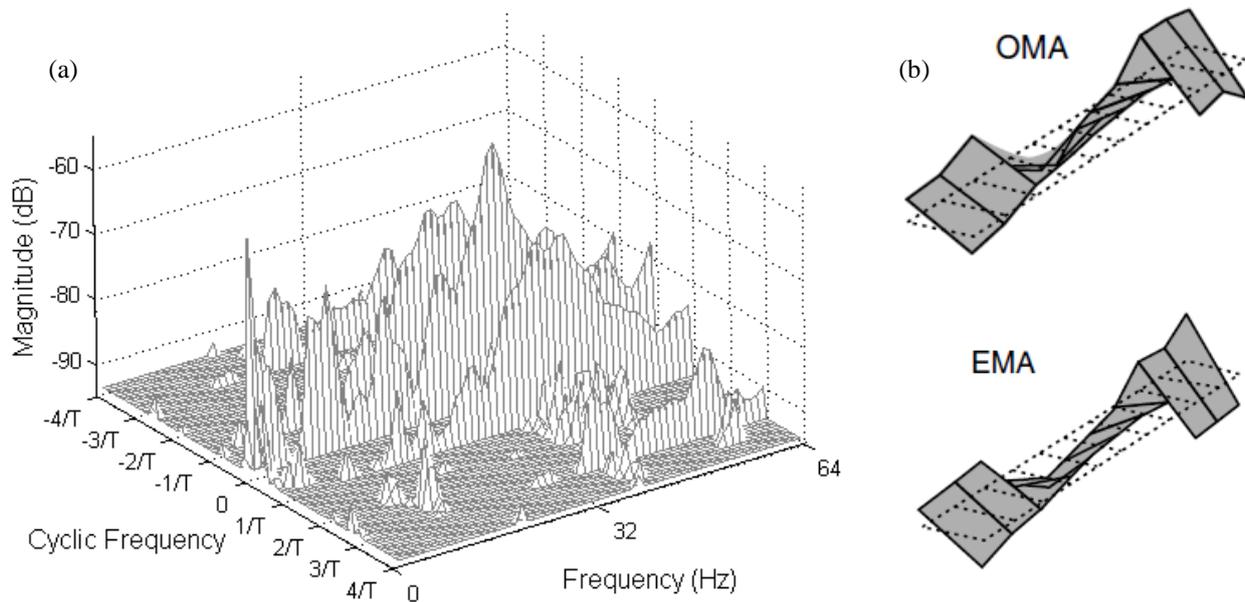


Figure 12: (a) Cyclic spectrum of a response measurement from a passenger rail vehicle [29]
(b) Comparison of a mode shape using cepstral OMA with experimental modal analysis (EMA)

5 Editing Time Signals using the Real Cepstrum

A new development, published after both [22] and [16], introduces the possibility of editing time signals using the real cepstrum. It was previously thought that this could only be done using the complex cepstrum, but as pointed out above this is not possible for stationary signals and so a large number of applications are excluded. It was realised [30] that there are many situations where editing could be carried out by modification of the amplitude only, which could be achieved using the real cepstrum. The modified amplitude spectrum can then be combined with the original phase spectrum of each record to generate an edited time record. A case in point is where discrete frequencies are to be removed. Whole families of uniformly spaced harmonics or sidebands can be removed by removing a small number of harmonics in the cepstrum. Removing a discrete frequency really means setting the value at that frequency to the expected value of the noise, of which the best guide are the frequency components on either side of the discrete

frequency, but usually at a much lower level. Setting the value of the discrete harmonics to zero automatically smooths over the amplitude of the log spectrum in the vicinity of the corresponding harmonics and/or sidebands (since notches in the log spectrum would equally give non-zero cepstrum components as for peaks) so at least the amplitude estimate would be correct. The phase in the modified spectrum at the position of the removed components would still be the same as that for the original discrete frequencies, and thus in general incorrect, but it should be kept in mind that these are typically spaced by 20 lines or more, and thus often negligible once their amplitude is reduced to that of the noise components, which in any case have random phase. The principle of the procedure is shown in Figure 13, where the original phase of each record is retained for combination with the modified amplitude obtained by liftering the real cepstrum.

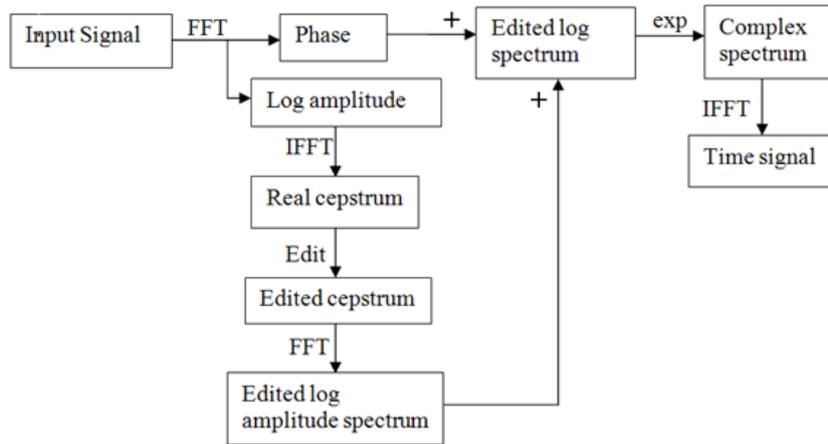


Figure 13: Schematic diagram of the cepstral method for removing selected components such as families of spectral harmonics and/or sidebands from time signals

Figure 14 shows a typical application to signals from a gearbox with a faulty bearing. It shows the original time signal, and compares it with the result of removing all harmonics of the gear signal by two methods (the gear ratio is 1:1 so only one set of harmonics had to be removed). The first method, Fig. 14(b), removed the Time Synchronous Average (TSA), while the cepstrum method results are shown in Fig. 14(c).

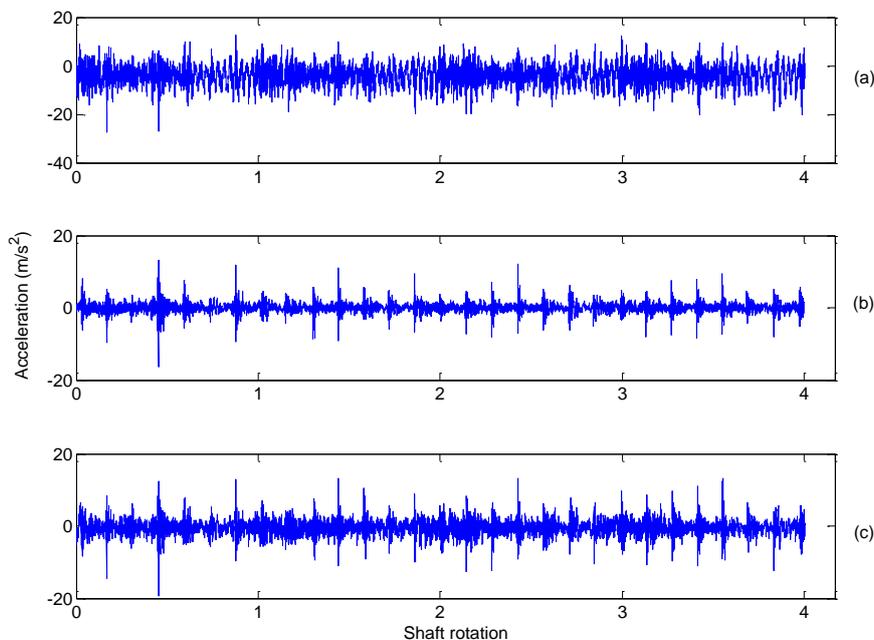


Figure 14: Time domain signals for gearbox test rig (a) Raw signal
 (b) Residual signal (after removing the synchronous average)
 (c) Residual signal after editing the Cepstrum to remove the shaft harmonics

Even though the result of the cepstral method is slightly noisier than for the TSA, the subsequent envelope analysis of the edited signals for the bearing diagnosis was just as successful.

Note that this cepstral method can also remove families of equally spaced sidebands, even where these are not simultaneously harmonics, something which cannot be done by TSA.

5.1 Application to operational modal analysis

A frequent problem with operational modal analysis (OMA) is the presence of disturbing components in the signal, for example discrete frequencies. These are often treated by the OMA software as modes with very low damping. Families of harmonics and sidebands can be individually removed, as just described, but another alternative that is often possible is to apply a lowpass lifter to the cepstrum to greatly enhance the modal information with respect to everything else. By comparing equations (9) and (15) it is seen that both contain damped exponentials, with the frequencies of the modes (and similar terms for the zeros), but the cepstrum is additionally weighted by $1/n$, making it shorter than the equivalent impulse response. Thus the information about all modes is concentrated at low quefrequency. Applying an exponential window (lowpass lifter) to the cepstrum removes almost all extraneous information (including discrete frequencies) at higher quefrequencies, and the only effect on the modal information is to add a precisely known amount of extra damping to each mode. This is the same in principle as using an exponential window in impact tests to cause impulse responses to die out before the end of the record, but can in this case be applied to stationary signals, rather than just transients starting at time zero.

Figure 15 shows an example where this has been applied to a signal from a high speed gas turbine, whose spectrum is dominated by multiple families of discrete harmonics. Figure 15(a) is the cepstrum of the original log spectrum 15(d), and 15(b) is an exponential lowpass lifter chosen to have a time constant of about 0.0025s, this being approximately equivalent to the damping of the lowest frequency mode (the lowest frequency peak in 15(e)). Thus, after applying this lifter, the damping of this lowest mode would be approximately doubled, but since the added bandwidth is constant it would have progressively less effect on high frequency modes. The damping of all modes can in any case be compensated.

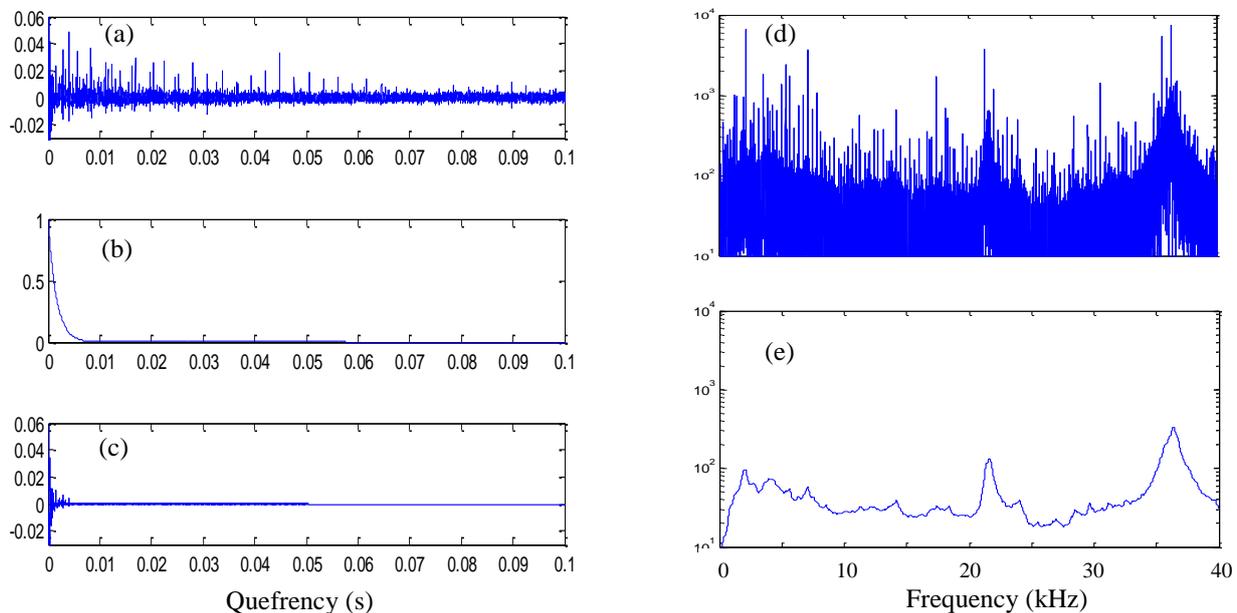


Figure 15 Use of an exponential lowpass lifter to enhance modal information (a) Full cepstrum (of (d)) (b) Exponential lifter (c) Liftered cepstrum (d) Original spectrum (e) Liftered spectrum

It will be seen that this almost blind operation has resulted in processed data almost completely dominated by the modal information (as characterised by the log spectrum in Fig. 15(e)). Even though only

the spectrum is shown, time records are available which could be processed by standard OMA software. Unfortunately, a modal analysis could not be carried out in this case as only one signal was available.

The same approach was however applied to the case of data from an in-flight helicopter, whose response signals were contaminated by the harmonics of the rotor speed. In this case, the damping of the lowest mode had a time constant of about 0.5s, which compared with 0.22s for the rotor period, so an exponential window could not completely remove the first two harmonics.

However, as shown in [31], the rotor harmonics could be largely removed, both by using a comb filter adjusted to the harmonic spacing, and also with an exponential filter on time constant 0.5s, but the best results were obtained using a combination of both. These were comparable with the results of a method using TSA to remove the rotor harmonics, but this was quite onerous because it required the signals first to be order tracked to compensate for very small speed variations.

6 Conclusion

Few people realise that the cepstrum is older than the FFT algorithm, or that its original definition made it less flexible as a result. Although originally applied to problems in seismology and speech analysis, similar background situations meant that it was applied quite early to mechanical problems as well. There are two main areas of applicability, which have been considerably developed over the years, one in detecting, classifying and perhaps removing families of harmonics and sidebands in vibration and acoustic signals, and the other in blind separation of source and transfer function effects, at least for SIMO systems. The latter leads naturally to applications in operational modal analysis. Despite its long history, the author believes that the cepstrum still has not been fully exploited, and it is hoped that this paper may stimulate further interest in its possibilities.

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