Modelling of tangential vibrations in cylindrical grinding contact with regenerative chatter

Veli-Matti Järvenpää, Lihong Yuan, Hessam Kalbasi Shiravani and Faisal Mehmood

Tampere University of Technology
Department of Engineering Design
P.O.Box 589, 33101 Tampere, Finland
{veli-matti.jarvenpaa, lihong.yuan, hessam.kalbasishiravani, faisal.mehmood}@tut.fi}

Abstract

The grinding contact of a cylindrical grinding process may have unstable vibration behaviour such as regenerative chatter or surface pattern generation. This will have strong effect on the surface quality of the work piece. The stability analysis of grinding contact has been studied in literature widely. Typically, the normal direction is considered. In this paper the tangential vibrations of the contact are considered by focusing to the rotational movements at the contact. A numerical model of the rotational vibration system is derived which includes the grinding wheel, shaft, belt transmission and motor subsystem and the work piece-motor subsystem. The normal and tangential cutting and sliding forces are coupled by the friction and the grinding penetration term and this leads to the description of the link between the tangential vibrations and the normal direction stability behaviour. In the results the time domain simulations and the stability conditions in two different running speed cases are presented.

1 Introduction

The cylindrical grinding is a manufacturing process to produce high quality surface finish on a cylindrical work piece surface. Grinding in general belongs to the material cutting processes such as turning and milling in which chips are removed by a machine tool. The vibration dynamics of these cutting processes is studied in literature widely. Some examples are for example: Thompson introduced the classical theory of chatter growth with double delays [1]; Moon explored the dynamical phenomena of several manufacturing processes [2]; Nayfeh considered the chatter control and the stability analysis under regenerative cutting conditions [3]; Altintas described the chatter vibrations in cutting with delay [4]; Stepan analysed cutting process stability by including double delay effects [5]; and Liu (et al) investigated double delay stability of cylindrical grinding [6].

The cylindrical grinding dynamics is characterized by the regenerative chatter phenomenon which is related to the time delay effects. The instability of the grinding process results from the contact interaction of the grindstone-work piece and may cause damage and failure such as unacceptable surface finish and pattern generation. The first chatter source is the work piece due to the overlap of the grinding path on its surface. A regenerative excitation source is generated to the system because the previous surface history is reintroduced to the grinding contact. The second chatter source is the grindstone itself because the surface of the grindstone also loses material slowly and these out-of roundness damages carry on by the rotational movement into the grinding contact interaction. Both of these chatter sources one from the work piece and another form the grindstone can be described as individual delay terms in the system dynamics modelling and equations.

The vibrations of the grinding system in the tangential direction are not considered usually because the obvious vibration behaviour clearly is in the normal direction, and the dynamical modelling at this direction provides sufficient equations to determine the stability conditions. The tangential direction, however, provides some additional information about the chatter vibration development in the grinding system. The main motivation to investigate the tangential direction is to increase the ability to observe the unstable chatter vibration phenomenon. In this paper the tangential direction is considered by modelling the rotational degrees-of-freedom system for the grinding vibrations.

2 Modelling of grinding system dynamics

The dynamics of cylindrical grinding system will be described now. The system equations will be presented at first and the grinding force together with the time delay terms will be presented in the next chapter. Figure 1 illustrates the rotational dynamics system and its members.

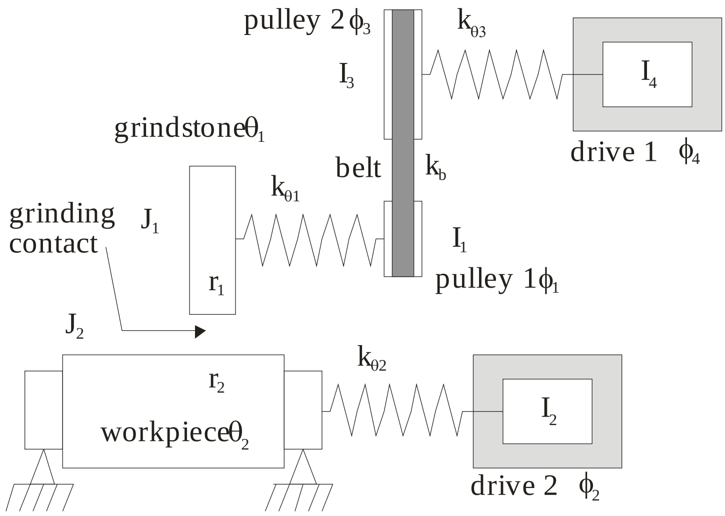

Figure 1: The rotational grinding system with the grindstone, the work piece, the shafts, the belt transmission and the drives.

The system equations can be written in the matrix form as

$$\begin{bmatrix} J_1 & & & & & \\ & I_1 & & & & \\ & & J_2 & & & \\ & & & I_2 & & \\ & & & & I_3 & \\ & & & & & I_4 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\phi}_1 \\ \ddot{\theta}_2 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \\ \ddot{\phi}_4 \end{bmatrix} \dots$$

$$\begin{bmatrix} k_{\theta 1} & -k_{\theta 1} & & & & \\ -k_{\theta 1} & k_{\theta 1} + 2k_b r_3 & & & -2k_b r_4 & \\ & & k_{\theta 2} & -k_{\theta 2} & & \\ & & -k_{\theta 2} & k_{\theta 2} & & \\ & -2k_b r_3 & & & k_{\theta 3} + 2k_b r_4 & -k_{\theta 3} \\ & & & & -k_{\theta 3} & k_{\theta 3} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \phi_1 \\ \theta_2 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} = \begin{bmatrix} -F_T r_1 \\ 0 \\ -F_T r_2 \\ T_{D2} \\ 0 \\ T_{D1} \end{bmatrix}, \tag{1}$$

where θ_i and ϕ_i are the rotational degrees of freedom, J_i and I_i the inertias, $k_{\theta i}$ the torsional stiffness, k_b the belt stiffness, r_i the radiuses and F_T the tangential grinding force (compare with Figure 1). The belt transmission system can be idealized by letting

$$I_{1red} = I_1 + i_{tr}^{\ 2} I_3 \tag{2}$$

where i_{tr} is the transmission ratio of the belt system. This leads to the more reduced system presented in Figure 2.

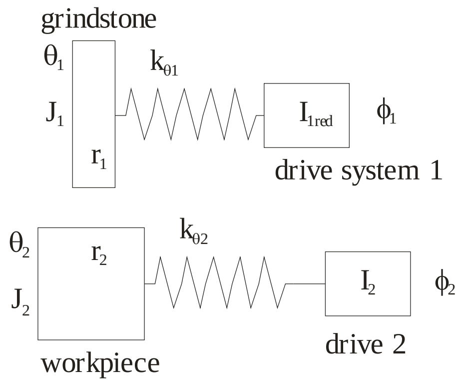

Figure 2: The reduced rotational grinding dynamic system.

The reduced system equations are

$$\begin{bmatrix} J_1 & & & \\ & I_{1red} & & \\ & & J_2 & \\ & & & I_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\phi}_1 \\ \ddot{\theta}_2 \\ \ddot{\phi}_2 \end{bmatrix} + \begin{bmatrix} k_{\theta 1} & -k_{\theta 1} & & \\ -k_{\theta 1} & k_{\theta 1} & & \\ & & k_{\theta 2} & -k_{\theta 2} \\ & & -k_{\theta 2} & k_{\theta 2} \end{bmatrix}\begin{bmatrix} \theta_1 \\ \phi_1 \\ \theta_2 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} -F_T r_1 \\ T_{D1} \\ -F_T r_2 \\ T_{D2} \end{bmatrix}. \tag{3}$$

Now, by introducing the variable transformation for the degrees of freedom of each shaft like

$$\begin{bmatrix} \theta_i \\ \phi_i \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -\dfrac{J_i}{I_i} \end{bmatrix}\begin{bmatrix} \theta_{Ri} \\ \Delta\phi_i \end{bmatrix} \tag{4}$$

the system equations take the forms

$$\begin{bmatrix} J_1 + I_{1red} & \\ & J_2 + I_2 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -F_T r + T_{D1} \\ -F_T r_2 + T_{D2} \end{bmatrix} \tag{5}$$

and

$$\begin{bmatrix} J_1 + \dfrac{J_1^{\,2}}{I_{1red}} & \\ & J_2 + \dfrac{J_2^{\,2}}{I_2} \end{bmatrix}\begin{bmatrix} \Delta\ddot{\phi}_1 \\ \Delta\ddot{\phi}_2 \end{bmatrix} + \begin{bmatrix} k_{\theta 1}\dfrac{(J_1 + I_{1red})^2}{I_{1red}^{\,2}} & \\ & k_{\theta 2}\dfrac{(J_2 + I_2)^2}{I_2^{\,2}} \end{bmatrix}\begin{bmatrix} \Delta\phi_1 \\ \Delta\phi_2 \end{bmatrix} = \begin{bmatrix} -F_T r + \dfrac{J_1}{I_{1red}} T_{D1} \\ -F_T r_2 + \dfrac{J_2}{I_2} T_{D2} \end{bmatrix}. \tag{6}$$

Finally, the vibrations at the normal direction can be described by 2-degrees-of-freedom model presented in Figure 3. The system equations in the normal direction are

$$\begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 & \\ & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 & \\ & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -F_N \\ F_N \end{bmatrix} \tag{7}$$

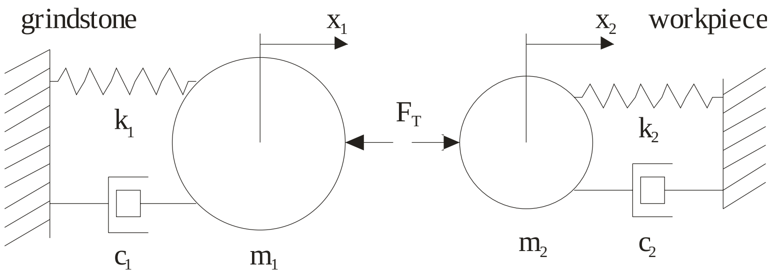

Figure 3: The grinding dynamic system in the normal direction.

where x_i are the translational degrees of freedom, m_i the masses, c_i the damping coefficients, k_i the stiffness, and F_N the normal grinding force. The complete system dynamics equations consists of (5), (6) and (7).

3 Grinding contact forces

The grinding forces will be described now. The normal and tangential grinding forces of the surface grinding case presented in [7] and [8] are used. It is assumed that it provides reliable accuracy and the equations can be updated to correspond to the cylindrical geometry in future contexts if necessary.

3.1 Surface grinding forces

In the normal direction one can describe the cutting penetration in the work piece at the following way (Figure 4) [9]. Let ε be the total penetration and the parameter β defines the material removal ration between the work piece and the grindstone. Thus, the total material removed is $\beta\varepsilon$ and β being close value to unity. In the tangential direction the chip removal regions can be divided to cutting (chip formation) and sliding zones. The force equation composition differs in these two zones due to the nature of cutting process at each zone and this lead to two terms in the grinding force equation in the normal direction. The zone between them is not considered. The tangential grinding force is related to the normal force by friction.

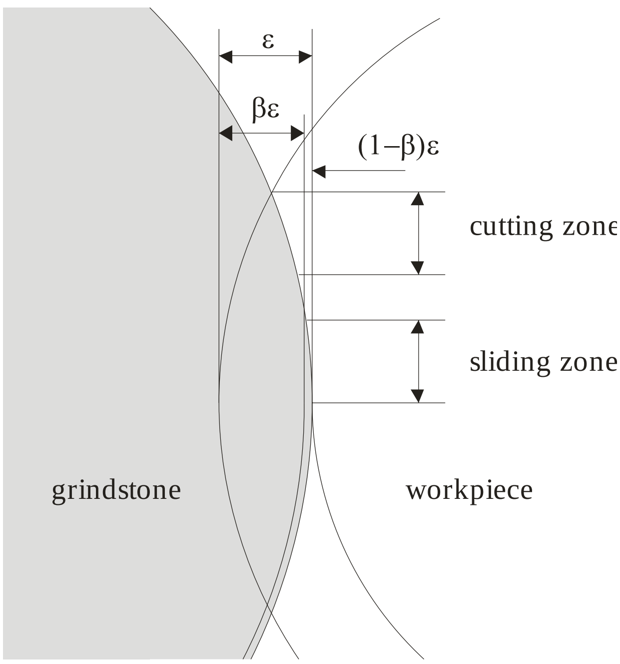

Figure 4: The normal direction penetration in the grinding contact and the cutting and sliding zones.

Thus, the normal and tangential grinding forces are

$$F_N = F_{Nc} + F_{Ns} \tag{8}$$

$$F_T = F_{Tc} + F_{Ts} \tag{9}$$

where F_{Nc} , F_{Ns}, F_{Tc} and F_{Ts} are the normal and tangential cutting and sliding force terms. These are expressed as

$$F_{Nc}(\varepsilon) = \{K_1 + K_2 \ln\left(\frac{v_G^{\frac{3}{2}}}{v_w^{\frac{1}{2}}} \frac{1}{\varepsilon^{\frac{1}{4}}}\right)\} \frac{v_G}{v_w} b \cdot \varepsilon \tag{10}$$

$$F_{Tc}(\varepsilon) = \{K_3 + K_4 \ln\left(\frac{v_G^{\frac{3}{2}}}{v_w^{\frac{1}{2}}} \frac{1}{\varepsilon^{\frac{1}{4}}}\right)\} \frac{v_G}{v_w} b \cdot \varepsilon \tag{11}$$

$$F_{Ns}(\varepsilon) = \overline{p}Ab\sqrt{b\varepsilon} \tag{12}$$

$$F_{Ts}(\varepsilon) = \mu \cdot F_{Ns} \tag{13}$$

where K_i are the experimental grinding force coefficients, v_G and v_w the contact velocities of the grindstone and the work piece, b is the width of the grindstone. Parameters $\overline{p}A$ are related to the sliding friction terms and μ is the friction coefficient. Because of the separate force terms in (8) and (9) one can into account only desired ones and investigate their effects individually in the simulations.

3.2 Time delay terms

The grindstone is moving horizontally on the surface of the work piece. The grinding path overlaps itself at some degree because this ensures that the whole surface will be ground. This horizontal movement specified two zones in which the grinding force with the delay effects are described as illustrated in Figure 5.

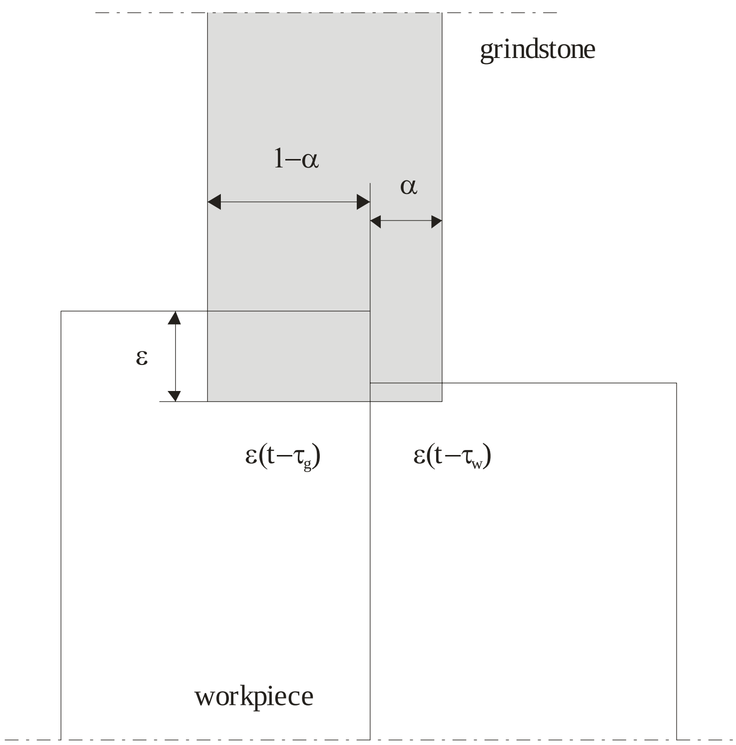

Figure 5: The overlap of the grinding path and the two different force zones.

The parameter α specifies the overlap ratio. Firstly, the fresh cutting zone lies on the work piece side and there is no delay term from this zone. Secondly, on the overlap surface area the delay effect is generated by the delay of the work piece τ_w. Finally, from the whole grindstone surface are the delay effect is generated by the delay of the grindstone τ_g. These three effects compose the total grinding forces with delays by substituting the penetration values to equations (10) to (13) as

$$F_N = (1-\alpha)F_N(\varepsilon_{nom} + \Delta\varepsilon) - \alpha\beta F_N(\Delta\varepsilon(t-\tau_w)) - (1-\beta)F_N(\Delta\varepsilon(t-\tau_g)), \tag{14}$$

$$F_T = (1-\alpha)F_T(\varepsilon_{nom} + \Delta\varepsilon) - \alpha\beta F_T(\Delta\varepsilon(t-\tau_w)) - (1-\beta)F_T(\Delta\varepsilon(t-\tau_g)), \tag{15}$$

where

$$\Delta\varepsilon = x_1 - x_2 \tag{16}$$

and ε_{nom} is the nominal cut depth for the grinding.

4 Methods and analysis

The time integration method used in the simulations in MATLAB® is described in [10]. The method belongs to the Newmark time integrator family and it is also known as the average constant acceleration method. The method is implicit, uses a predictor-corrector approach and it includes the Newton-Rhapson iteration procedure. The iteration matrix used is

$$\mathbf{S} = \mathbf{K} + \frac{\gamma_I}{\beta_I h}\mathbf{C} + \frac{1}{\beta_I h^2}\mathbf{M}, \tag{17}$$

where β_I and γ_I are the time integration coefficients and h is the time step. The model also has a small Rayleigh damping with the damping matrix

$$\mathbf{C} = \alpha_R\mathbf{M} + \beta_R\mathbf{K}, \tag{18}$$

where α_R and β_R are damping coefficients. A PD-speed controller is defined by

$$T_{Di} = K_P(\dot{\phi}_d t - \phi_i) + K_D(\dot{\phi}_d - \dot{\phi}_i), \tag{19}$$

where $\dot{\phi}_d$ is the desired rotation speed and K_P and K_D the control coefficients.

The time integration procedure must be upgraded for the time delay equation. The method of steps can be used for the time domain solution of time delay equations [11]. For a delay differential equation an initial history period is required as a start up for the delay effects to develop. The method of steps procedure specifies an initial function on interval of [t_0-τ , t_0], where τ is the delay time (Figure 6). This so-called initial period of the delay equation is solved without delay effects at the beginnig. Then the delay differential equation can be solved by taking the delay term values from the history .

In the case of rotational vibrations the time delays are non-constant and they are directly related to the rotation speeds of the grindstone and the work piece. This requires more complex solution procedure because the exact values of the delayed variables are not directly available from the data of previous time increments due to the discrete solution. The values must be estimated by a two-step polynomial interpolation procedure [8]. In the first step, the unknown value of the time delay is determined from the inverse curve of the rotation angle θ_i by subtracting 2π from its current value and then interpolating the delayed time $t-\tau$. In the second step the unknown values of the delayed variables are interpolated according to the values of the delay times. This procedure is implemented in a code but it is possible to introduce it for example in the SIMULINK® environment.

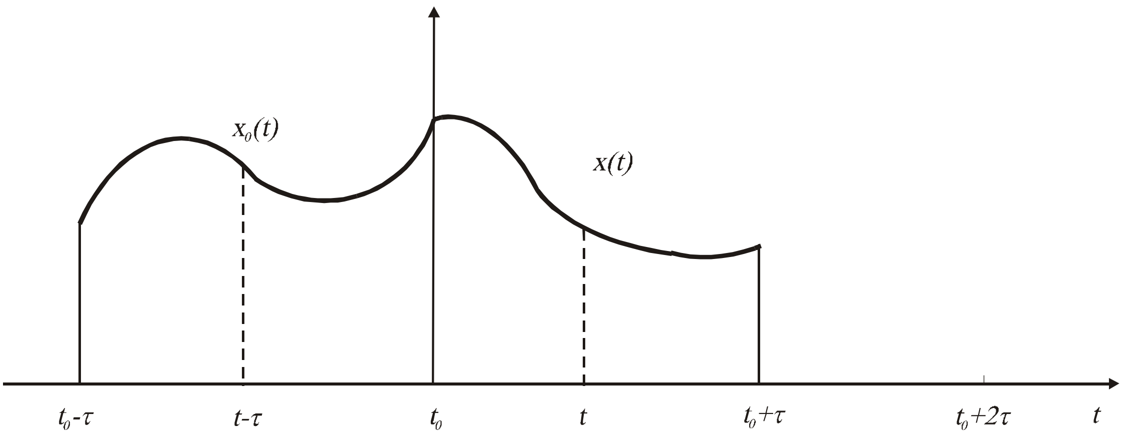

Figure 6: The initial history period $x_0(t)$ in the method of steps procedure.

The general objective of the analysis in this paper is to illustrate the stability characteristics of the six degrees-of-freedom grinding system. It is well known that the system has multiple unstable regions depending on the values of the key system parameters. As discussed in the introduction this topic has been considered in the literature widely. Typical parameters defining the stability are the running speeds (the time delay values) and for example the stiffness of the grinding contact. The primary objective is to investigate how the coupling of the normal and tangential grinding forces influences to the vibrations in the rotational direction. The unstable behaviour will develop at the normal direction and the vibrations reflecting this at the tangential direction are at the interest.

5 Numerical results and discussion

The numerical example is created based on the parameters from a heavy industrial grinding machine used for the grinding of large cylindrical objects made from steel. The mass of the grindstone is 250 kg and the work pieces can have masses up to 4000 kg. The horizontal lowest natural frequency of the grindstone mount is about 200 Hz and the work piece is long cylinder with natural frequency between 10 to 20 Hz. Radiuses of the grindstone and the work piece are 250 mm and about 200 mm. The width of the grindstone is 80 mm. The running speeds are 20-30 Hz and less than 1 Hz, respectively. The average cutting depth is 20 μm and the parameters K_1 and K_3 in (10) and (11) are about $300 \cdot 10^6$ N/m^2.

With these system parameters a stability boundary can be found between the grindstone tangential contact velocities of 25 m/s and 30 m/s. Thus, two simulation cases were chosen to illustrate the vibration behaviour of the grinding system. At the running velocity 20 m/s (about 13 Hz) the system is stable and at 35 m/s (about 22 Hz) it is unstable. Only the cutting force terms were considered in the simulations. Figure 7 shows the normal force and the work piece rotational vibrations at the stable case. The vibrations due to the initiation of the grinding contact at the beginning are dying out when the grinding continues. The delay effect carries on in the simulation but it does not act in unstable fashion. The model has very modest viscous damping at the normal direction in the simulations, which is not large enough to cancel the delay excitation sources. Figure 9 illustrates the work piece motor dynamic behaviour. The bigger scale waviness in the figures is due to the PD speed control.

Figures 9 and 10 show the unstable case. Now the chatter vibration effect is developing in the normal direction clearly. Also at the tangential direction the chatter becomes detectable even though the scale of the effect seems to be at a smaller range. Important remark is that the PD speed control does not react to the tangential chatter vibrations because the control parameters were chosen in such way that the control behaves rather weakly. The aim is to investigate the delay effect phenomenon instead of controlling it. It is, however, obviously possible to use PI or PID control for the speed which can compensate the chatter vibrations at the tangential direction, but even in this case the chatter becomes detectable then from the control signals. The observation of the rotational degrees-of-freedom seems to provide a feasible method to detect the chatter vibrations in the cylindrical grinding system.

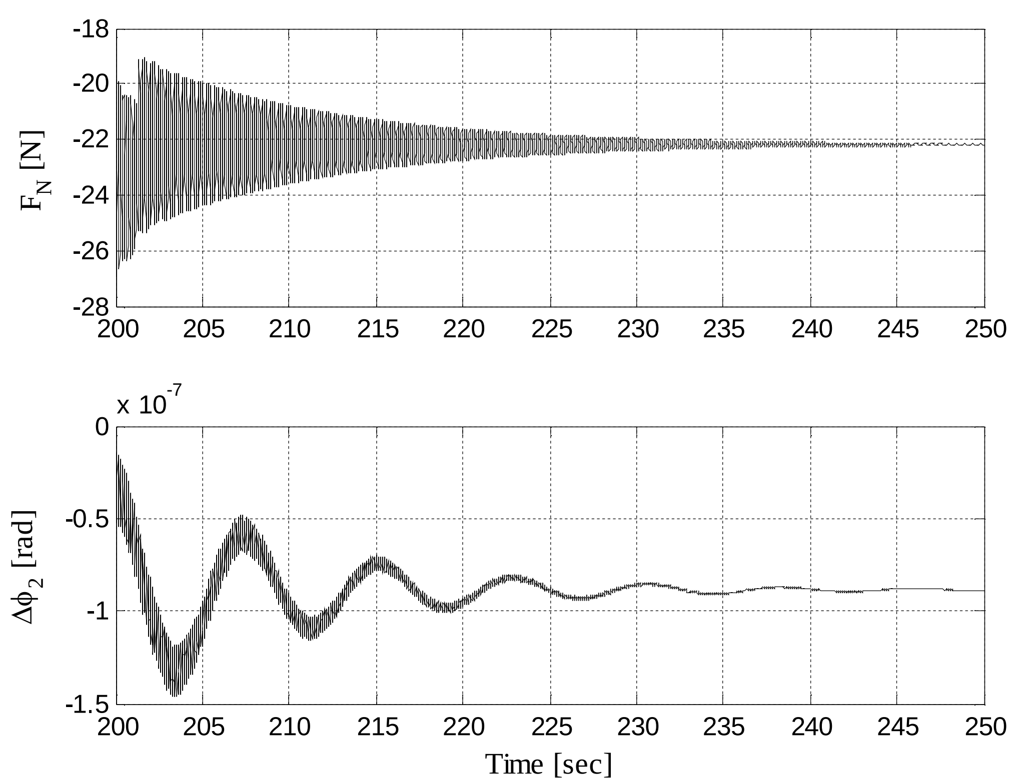

Figure 7: The stable case. The normal force is above and the rotational vibration of the work piece below. The grindstone's contact velocity is 20 m/s and the work piece's 1m/s.

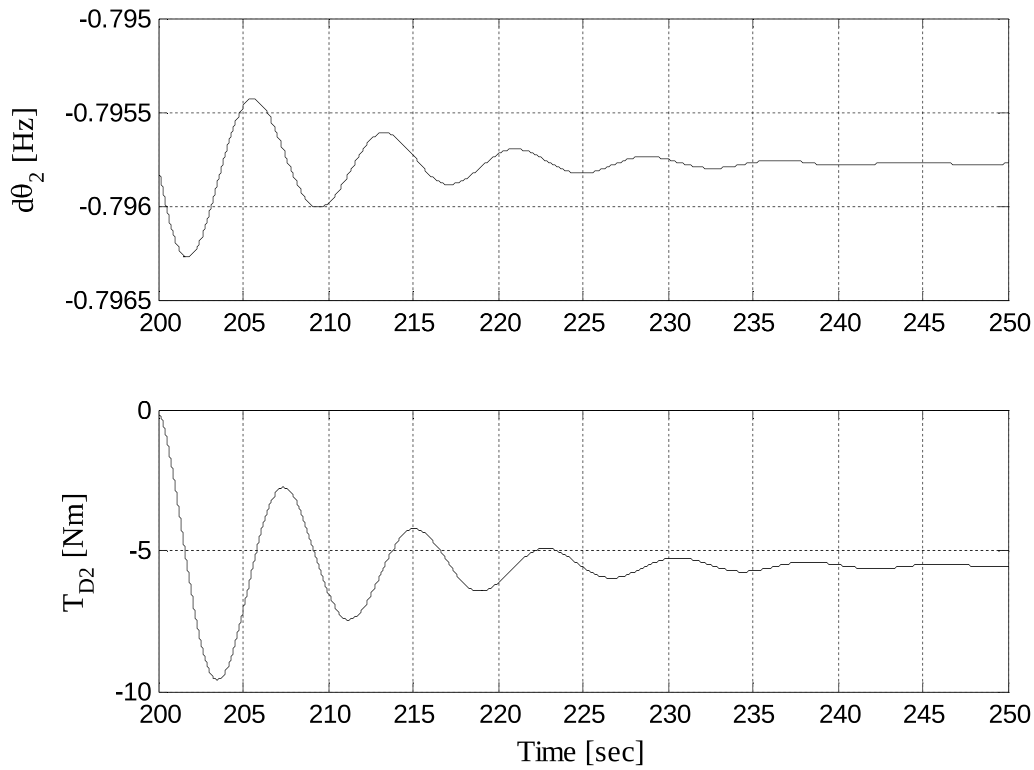

Figure 8: The stable case. The angular velocity of the work piece is above and the torque of the work piece motor below. The grindstone's contact velocity is 20 m/s and the work piece's 1m/s.

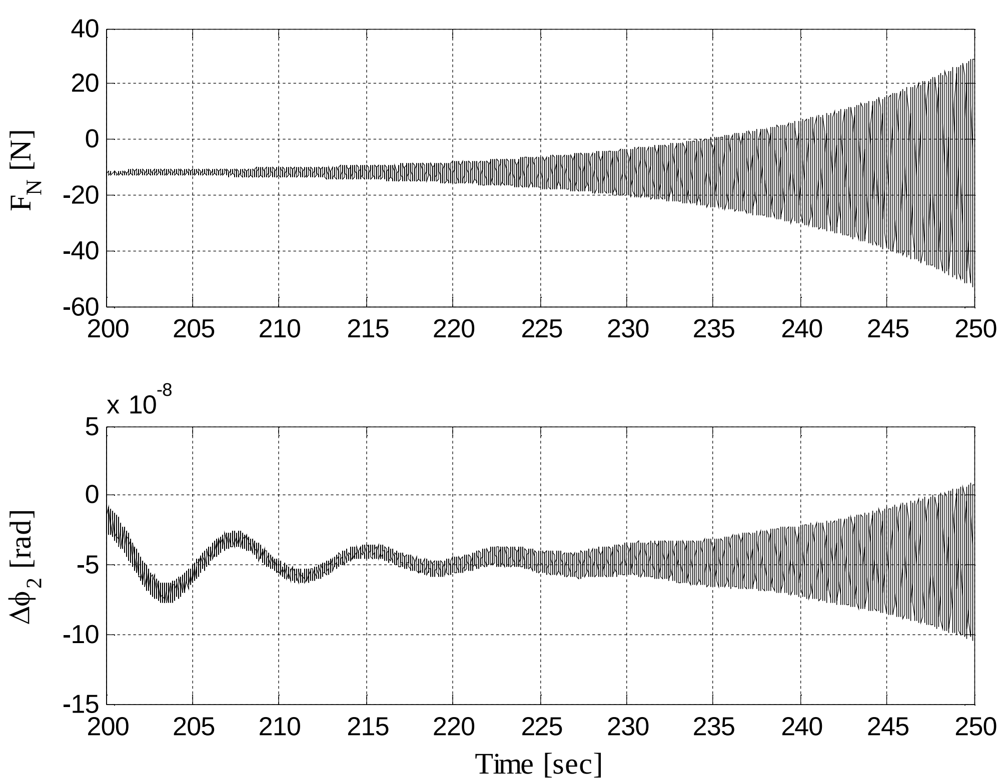

Figure 9: The unstable case. The normal force is above and the rotational vibration of the work piece below. The grindstone's contact velocity is 35 m/s and the work piece's 1m/s.

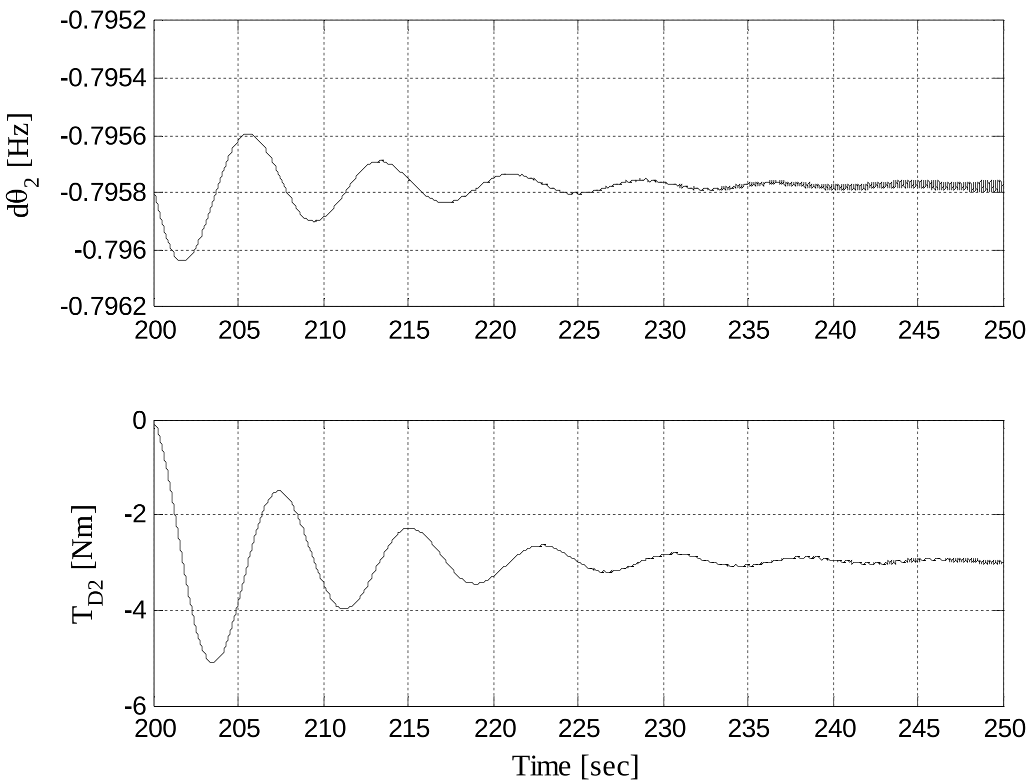

Figure 10: The unstable case. The angular velocity of the work piece is above and the torque of the work piece motor below. The grindstone's contact velocity is 20 m/s and the work piece's 1m/s.

Conclusions

The numerical solution of the delay equation seems to work well. The first main purpose of this solution procedure is to provide a tool for the stability analysis of complex delay systems. An alternative use could be for measurement data verification to detect the presence of delays in the system measured. Further work is required to verify the modelling approach of this paper.

Acknowledgments

The authors would like to express their gratitude to the Academy of Finland, which has provided funding for this reserach.

References

[1] R. A. Thompson, *On the Doubly Regenerative Stability of a Grinder: the Theory of Chatter Growth*, Journal of Engineering for Industry, May 1986, Vol. 108/75.

[2] F. C. Moon, *Dynamics and Chaos in Manufacturing Processes*, John Wiley & Sons, Inc., 1998

[3] A. H. Nayfeh, *Problems in Perturbation*, 1993

[4] Y. Altintas, *Manufacturing Automation*, Cambridge University Press, 2000.

[5] G. Stepan, *Modelling Nonlinear Regenerative Effects in Metal Cutting*, Phil Trans R Soc Lond A (2001) 359, The Royal Society, 2001

[6] A. Liu, G. Payre, *Stability Analysis of Doubly Regenerative Cylindrical Grinding Process*, Journal of Sound and Vibration, 301 (2007) 950-962

[7] J. Tang, J. Du, Y. Chen, *Modeling and Experimental Study of Grinding Forces In Surface Grinding*, Journal of Materials Processing Technology, 209 (2009) 2847-2854

[8] S. Malkin, *Grinding Technology Theory and Applications of Machining with Abrasives*, 2002

[9] L. Yuan, E. Keskinen, V. M. Järvenpää, *Stability Analysis of Roll Grinding System with Double Delay Effects*, IUTAM Symposium on Vibration Control of Nonlinear Mechanisms and Structures, Solid Mechanics and its Applications Volume 130, 2005, 375-387

[10] M. Géradin, D. Rixen, *Mechanical Vibrations-Theory and Application to Structural Dynamics*, John Wiley & Sons, Inc., 1997.

[11] R. D. Driver, *Ordinary and Delay Differential Equations*, Springer-Verlag, New York, 1997